۱۸/۲۵

مهران فراغتی - دهم دختر C - تکلیف شماره ۲۸

**۱**

$$f(x)=\begin{cases} x^2+2x & ; x\geq a \\ ax-4 & ; x\leq a \end{cases}$$

$f(a)=a^2+2a$

$f(a)=a^2-4$

$a^2+2a=a^2-4 \rightsquigarrow 2a=-4$

$\rightsquigarrow a=\frac{-4}{2}=-2 \rightsquigarrow \boxed{a=-2}$

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**۲**

$f(x)=\frac{x^2+a}{2x-b}$ , $g(x)=2x+b$ (۲,۳) متقاطع اند

$\rightsquigarrow f(2)=\frac{2^2+a}{2\times2-b}=\frac{4+a}{4-b}=3 \rightsquigarrow 4+a=12-3b$

$a=8-3b$

$g(2)=2\times2+b=4+b=3 \rightsquigarrow b=3-4=-1 \rightsquigarrow b=-1$

$a=8-3b=8-3(-1)=8+3=11 \rightsquigarrow a=11$

$f(x)=\frac{x^2+11}{2x+1} \rightsquigarrow f(1)=\frac{1^2+11}{2\times1+1}=\frac{1+11}{2+1}=\frac{12}{3}=4 \rightsquigarrow \boxed{f(1)=4}$

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**۳**

$f(x)=\frac{4x+1}{2x^2+ax+b}$ $D_f=\mathbb{R}-\{-1,4\}$

$\{-1,4\} \rightsquigarrow$ ریشه های مخرج هستند

$2(-1)^2+a(-1)+b=0 \rightsquigarrow 2-a+b=0$

$2(4)^2+a(4)+b=0 \rightsquigarrow 32+4a+b=0$

$2-a+b=32+4a+b \rightsquigarrow -30=5a \rightsquigarrow a=-6$

$\rightsquigarrow 2-(-6)+b=0 \rightsquigarrow 8+b=0 \rightsquigarrow b=-8$

$f(x)=\frac{4x+1}{2x^2-6x-8} \rightsquigarrow f(1)=\frac{4+1}{2-6-8}=\frac{5}{-12} \rightsquigarrow \boxed{f(1)=-\frac{5}{12}}$

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**۴**

$f(x)=\frac{x^3-\sqrt{3}}{-4x^2+ax+b}$ $D_f=\mathbb{R}-\{-1\}$

$\{-1\} \rightsquigarrow$ ریشه مضاعف ریشه است

$(x+1)^2=x^2+2x+1 \xrightarrow{\times(-4)} -4x^2-8x-4$

$\rightsquigarrow -4x^2+ax+b=-4x^2-8x-4$

$ax+b=-8x-4 \rightsquigarrow a=-8$ , $b=-4$

$\rightsquigarrow a+b=-8-4=-12 \rightsquigarrow \boxed{a+b=-12}$

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**۵**

$f(x)=\frac{2x}{(x-1)(x^2+mx+1)}$ $D_f=\mathbb{R}-\{1\}$

$\rightsquigarrow x^2+mx+1 \rightsquigarrow$ ریشه ندارد

$b^2-4ac<0 \rightsquigarrow m^2-4\times1\times1<0 \rightsquigarrow m^2-4<0$

$\rightsquigarrow m^2<4 \rightsquigarrow -2<m<2 \rightsquigarrow m=(-2,2)$

$\rightsquigarrow x^2+mx+1 \rightsquigarrow$ ریشه مضاعف ۱ دارد

$(x-1)^2=x^2-2x+1 \rightsquigarrow \cancel{x^2}+mx+\cancel{1}=\cancel{x^2}-2x+\cancel{1}$

$\rightsquigarrow mx=-2x \rightsquigarrow m=-2 \rightsquigarrow \boxed{m=[-2,2)}$

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**۶**

۰/۲۵

$f(x)=\sqrt{4-\frac{1}{x^2}} \rightsquigarrow 4-\frac{1}{x^2}\geq0 \rightsquigarrow \cancel{4\geq\frac{1}{x^2}}$

$\rightsquigarrow \frac{1}{4}\leq x^2 \rightsquigarrow -\frac{1}{2}\leq x\leq\frac{1}{2} \rightsquigarrow \boxed{x=[-\frac{1}{2},\frac{1}{2}]}$

$\frac{4x^2-1}{x^2}\geq0$ $4x^2-1\geq0$ $x^2\geq\frac{1}{4}$

$x\geq\frac{1}{2} \rightarrow x\in(-\infty,-\frac{1}{2}]\cup[\frac{1}{2},+\infty)$

$x\leq-\frac{1}{2}$

## ۷

$f(x) = \sqrt{mx^2 + 2mx + 1} \rightsquigarrow D_f = \mathbb{R}$

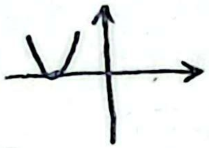
$a > 0$

$m > 0$

$mx^2 + 2mx + 1 \rightsquigarrow$ ریشه‌ی مضاعف دارد $\rightsquigarrow$ $+ \quad 0 \quad +$

$\rightsquigarrow b^2 - 4ac = 0 \rightsquigarrow (2m)^2 - 4(m)(1) = 0 \rightsquigarrow 4m^2 - 4m = 0$

$\rightsquigarrow m^2 - m = 0 \rightsquigarrow m(m-1) = 0 \rightsquigarrow m = \{0, 1\}$

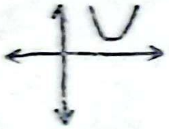


$mx^2 + 2mx + 1 \rightsquigarrow$ ریشه نداشته باشد $\rightsquigarrow$ $+$

$b^2 - 4ac < 0 \rightsquigarrow (2m)^2 - 4(m)(1) < 0 \rightsquigarrow 4m^2 - 4m < 0$

$\rightsquigarrow 4m^2 < 4m \rightsquigarrow m^2 < m \rightsquigarrow m < 1$

$\rightsquigarrow m > 0$ $(a > 0)$ $\rightsquigarrow$



$(-\infty, 1) \cap (0, +\infty) = (0, 1)$

$\rightsquigarrow m = \{0, 1\} \cap \; 0 < m < 1 \rightsquigarrow \boxed{0 \le m \le 1 \quad m = [0, 1]}$

## ۸

$f(x) = \begin{cases} \dfrac{4x^2 - 1}{2x - 1} & ; x \neq a \\ 4x + K & ; x = \frac{1}{2} \end{cases} \rightsquigarrow D_f = \mathbb{R} \quad , \quad g(x) = 2x + 1 \rightsquigarrow D_g = \mathbb{R}$

$2x - 1 = 0 \rightsquigarrow 2x = 1 \rightsquigarrow x = \frac{1}{2}$

$\rightsquigarrow x \neq \frac{1}{2} \rightsquigarrow a = \frac{1}{2}$

$g(\frac{1}{2}) = 2 \times \frac{1}{2} + 1 = 1 + 1 = 2$

$2 + K = 2 \rightsquigarrow K = 0$

$f(\frac{1}{2}) = 4 \times \frac{1}{2} + K = 2 + K$

$\rightsquigarrow a + K = \frac{1}{2} + 0 = \boxed{\frac{1}{2}}$

## ۹

$f(x) = \begin{cases} \dfrac{9x^2 - 4}{3x + 2} & ; x \neq -\frac{2}{3} \\ 3ax + 2 & ; x = -\frac{2}{3} \end{cases} \quad , \quad g(x) = 3x + b \rightsquigarrow D_g = \mathbb{R}$

$f(1) = \dfrac{9 - 4}{3 + 2} = \dfrac{5}{5} = 1 \rightsquigarrow D_f = \mathbb{R}$

$g(1) = 3 + b$

$3 + b = 1 \rightsquigarrow b = -2$

$f(-\frac{2}{3}) = 3 \times (-\frac{2}{3}) \times a + 2 = -2a + 2$

$g(-\frac{2}{3}) = 3(-\frac{2}{3}) + b = -2 - 2 = -4$

$\rightsquigarrow -2a + 2 = -4 \rightsquigarrow -2a = -6 \rightsquigarrow a = 3$

$a - b = 3 - (-2) = 3 + 2 = 5 \rightsquigarrow \boxed{a - b = 5}$

## ۱۰

$f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & ; x \neq 2 \\ 2a^2 + ax & ; x = 2 \end{cases} \quad , \quad g(x) = x + 2$

$f(2) = 2a^2 + 2a \rightsquigarrow 2a^2 + 2a = 4$

$g(2) = 2 + 2 = 4 \rightsquigarrow a^2 + a = 2$

$\rightsquigarrow a^2 + a - 2 = 0 \rightsquigarrow 1a^2 + 1a - 2 = 0$

$b^2 - 4ac = 1^2 - 4 \times 1(-2) = 1 + 8 = 9$

$\dfrac{-b \pm \sqrt{\Delta}}{2a} = \dfrac{-1 \pm \sqrt{9}}{2} = \dfrac{-1 \pm 3}{2} \rightsquigarrow \dfrac{-4}{2} = -2$

$\rightsquigarrow \dfrac{2}{2} = 1$

$\boxed{a = \{-2, 1\}}$