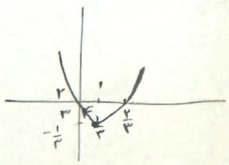
فارسی ریاضی . دهم اختر ج . تألیف شماره ۲۵ و ۶

① الف) $y = 3x^2 - 2x$ ← سهمی دهانه رو به بالا ← $c=0$ ← از مبدأ می‌گذرد

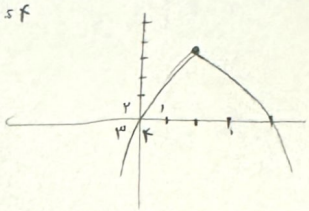
$x_s = \frac{-b}{2a} = \frac{2}{6} = \frac{1}{3}$ $\quad y_s = \frac{-4}{12} = -\frac{1}{3}$ $\quad$ رئوس $= (\frac{1}{3}, -\frac{1}{3})$



ب) $y = -x^2 + 4x$ ← سهمی دهانه رو به پایین

$y = x(-x+4)$ ← ریشه‌ها 0 و 4

$x_s = \frac{-4}{-2} = 2$ $\quad y_s = -4 + 8 = 4$ $\quad$ رئوس $= (2, 4)$

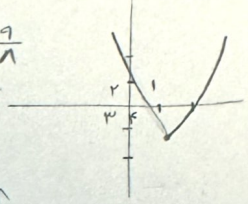


از ناحیه دوم نمی‌گذرد

② $\Delta = 25 - 16 = 9$

الف) $y = 2x^2 - 5x + 2$ ← ضرب ریشه‌ها

$x_s = \frac{5}{4}$ $\quad y_s = \frac{-9}{8}$



$x=2 \Rightarrow y=0$

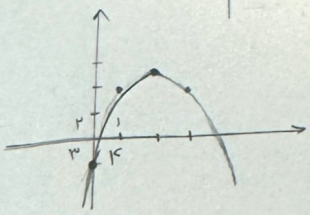
ب) $y = -x^2 + 4x - 1$

$x_s = \frac{-4}{-2} = 2$ $\quad y_s = -4 + 8 - 1 = 3$

$-1+4-1 < 2$

$-9+12-1 < 2$

x	1	2	3
y	2	3	2



از ناحیه اول، سوم و چهارم نمی‌گذرد

از ناحیه اول و دوم و چهارم می‌گذرد

③ $2x^2 - x - 3 = 0$ $\quad \alpha, \beta \Rightarrow$ ریشه‌ها

مجموع ریشه‌ها $= \frac{-b}{a}$ $\quad$ حاصل‌ضرب ریشه‌ها $= \frac{c}{a}$ $\quad$ تفاضل ریشه‌ها $= \frac{\sqrt{\Delta}}{|a|}$

الف) $\frac{\alpha+\beta}{\alpha-\beta} = \frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13}$

$\alpha+\beta = \frac{1}{1} = 1$ $\quad \alpha-\beta = \frac{\sqrt{1+12}}{1} = \sqrt{13}$

ب) $\alpha^2+\beta^2 = S^2 - 2P = 1 - 2(-3) = 1+6 = 7$

$P = \frac{c}{a} = -3$

ج) $\alpha^3+\beta^3 = S^3 - 3PS = 1 - 3(-3)(1) = 1+9 = 10$

د) $\alpha^3-\beta^3 = (\alpha-\beta)(\alpha^2+\beta^2+\alpha\beta) = \sqrt{13}(4) = 4\sqrt{13}$

الف ← $\sqrt{13}$ $\quad$ ب ← 7 $\quad$ ج ← -3

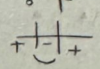
④ $y = (x-2)(x^2 - ax + a)$ ← این عبارت یک ریشه دارد.

حالت ۱) ← ریشه‌ها 2 ← $(x^2 - ax + a) = (x-2)^2 = (x^2 - 4x + 4)$ $\quad \Rightarrow a = 4$

حالت ۲) ← دارای دو ریشه متمایز ← $\Delta > 0$ $\quad x^2 - ax + a$ $\quad \Delta = b^2 - 4ac$

$\Delta = a^2 - 4a > 0$

$a(a-4) > 0$



$a = (0, 4]$

⑤ $3x^2 - 12x - a = 0$ $\quad \alpha, \beta \Rightarrow$ ریشه‌ها $\quad x = \alpha$

$3\alpha^2 - 12\alpha - a = 0 \Rightarrow \alpha^2 - 4\alpha = \frac{a}{3}$

$2\alpha^2 + \beta^2 - 4\alpha = 7$

$\alpha^2 + \beta^2 = S^2 - 2P = 16 - 2(-\frac{a}{3}) = 16 + \frac{2a}{3}$

$(\alpha^2 - 4\alpha) + (\alpha^2 + \beta^2) = 7 \Rightarrow \frac{a}{3} + \frac{2a}{3} + 16 = 7 \Rightarrow a + 16 = 7 \Rightarrow a = -9$

$3x^2 - 12x + 9 = 0$

مجموع ریشه‌ها $\Rightarrow \begin{cases} x=1 \\ x = \frac{c}{a} = \frac{9}{3} = 3 \end{cases}$ $\quad$ ریشه بزرگ‌تر $= 3$

جواب $\frac{-9}{10} = -3$

⑨ $x,y \in \mathbb{N}$

$A(2a+3, a-2) \rightsquigarrow 2a+3>0 \rightarrow 2a>-3 \rightarrow a>-\frac{3}{2}$, $a-2>0$, $a>2$

$B(7-2a, a-2) \rightsquigarrow 7-2a>0 \rightarrow -2a>-7 \rightarrow a<3/5$

$\Rightarrow a=3$

$A(9,1)$ $B=(1,1)$ $S(b, b-2)$

$x_S = \frac{9+1}{2} = \frac{10}{2} = 5 = b$ $\Rightarrow S=(5,3)$

$y = a(x-5)^2+3$ (ضریب x^2 برابر $-\frac{1}{8}$)

$(1,1)$ $a(-4)^2+3=1$ $\rightarrow 16a=-2 \rightarrow a=-\frac{1}{8}$

عرض از مبدا $= c = -\frac{25}{8}+3 = -\frac{1}{8}$

جواب $\Rightarrow \{1\}$ و چون فاصله

⑦ امتیاز رشته ها

$ax^2-ax-b=0 \rightsquigarrow \alpha, \beta$

$S = -\frac{b}{a} = \frac{a}{a} = 1$

$\alpha+\beta=1 \rightarrow \beta = 1-\alpha$

$B=1-\alpha$ $(4\beta^2+2\alpha^2-3\beta)=17$

$4(1-\alpha)^2+2(\alpha^2)-3(1-\alpha)=17$

$4+4\alpha^2-8\alpha+4-2\alpha^2-3+3\alpha=17$

$2\alpha^2-5\alpha+5=17$

$2\alpha^2-5\alpha+5=0$ $\rightsquigarrow 2\alpha^2-5\alpha-12=0$

$\Delta = b^2-4ac = 25-4(2)(-12) = 121$

$\alpha = \frac{-b\pm\sqrt{\Delta}}{2a} = \frac{5\pm\sqrt{121}}{4} = \frac{5\pm 11}{4}$

(1) $\rightsquigarrow \alpha = \frac{5+11}{4} = 4 \rightarrow \beta = 1-4 = -3$

$\alpha = \frac{5-11}{4} = -\frac{3}{2} \rightarrow \beta = 1+\frac{3}{2} = \frac{5}{2}$

اختلاف $|\alpha-\beta| = \left|-\frac{3}{2}-\frac{5}{2}\right| = |-4| = 4$

جواب

⑧ $(-3,\beta)$ و $(1,\beta)$

$x_S = \frac{-3+1}{2} = -1$

$y_S = -\frac{1}{2}$

$y = a(x+1)^2 - \frac{1}{2}$

$y = ax^2+2ax+a-\frac{1}{2}$

$a-\frac{1}{2} = 1+\frac{3}{2} \rightarrow a = 2 \rightarrow a = \frac{1}{2}$

$y = \frac{1}{2}x^2+2x+\frac{3}{2}$

$x=1$ $\rightsquigarrow \frac{1}{2}+2+\frac{3}{2} = 2+2 = 4$ $\rightsquigarrow \beta = 4$

$S=-4$ $P=a$ $\Delta = 4^2-4a$

⑨ $ax^2+4x+a=0$ $\alpha<\beta<0$ $a=?$

$\sqrt{4(9-a)} = 2\sqrt{9-a}$

$\alpha = \frac{-4+\sqrt{4^2-4a}}{2}$ $\beta = \frac{-4-\sqrt{4^2-4a}}{2}$

$\alpha = -2-\sqrt{9-a}$ $\beta = -2+\sqrt{9-a}$

$3\alpha^2+2\beta^2 = 12\sqrt{2}+18$

$\alpha^2+(\alpha^2+\beta^2) = 12\sqrt{2}+18$

$S^2-2P = 2(4^2-4a)+(-2-\sqrt{4a})^2 = 32-8a+\underbrace{4+9-a}_{13}+4\sqrt{9-a} = \underbrace{12\sqrt{2}}_{3\times2\times2}+18$

$9-a=1 \rightarrow (a=8)$

⑩ $3x^2-(m+14)x+1=0$

$\frac{1}{\sqrt{\alpha}}+\frac{1}{\sqrt{\beta}} = \frac{\sqrt{\beta}+\sqrt{\alpha}}{\sqrt{\alpha\beta}} = \frac{\sqrt{\beta}+\sqrt{\alpha}}{\frac{1}{3}} = 3(\sqrt{\beta}+\sqrt{\alpha}) = 3\sqrt{m+26} = 9 \rightarrow m+26=9 \Rightarrow m=-17$

$(\sqrt{\alpha}+\sqrt{\beta})^2 = \alpha+\beta+2\sqrt{\alpha\beta} = \frac{m+14}{3}+\frac{2}{3} = \frac{m+26}{3}$ $\rightsquigarrow \sqrt{\alpha}+\sqrt{\beta} = \frac{\sqrt{m+26}}{3}$

$y = mx^2+2x+2$ $\xrightarrow{m=-17}$ $y = -17x^2+2x+2$ $P = \frac{c}{a} = \frac{2}{-17}$