

1 /

Exercise 1

$$\begin{aligned} -b &= 1 \quad b=1 \quad x=1 \quad a+b+c=9 \quad a+c=9 \quad x=r \\ 9a+rb+c &= 1 \quad 9a=1 \quad a=\frac{1}{9} \quad b=r(-\frac{1}{9})=1 \quad c=9-\frac{1}{9}-1 \\ y &= \frac{1}{9}x^2 - x + 1 \end{aligned}$$

$$\begin{aligned} y &= rx^2 + mx + m + 4 \geq 0 \quad \text{substituting } r \\ m^2 - f(4m+1) &\geq 0 \quad m^2 - 4m - f \geq 0 \\ (m+f)(m-4) &\geq 0 \quad \frac{-f \pm \sqrt{f^2 + 16}}{2} \\ \textcircled{1} m \in (-\infty, -f) \cup (4, +\infty) \quad \textcircled{2} \delta = -\frac{m}{r} &\geq 0 \rightarrow m \leq 0 \quad \textcircled{3} \frac{m+4}{r} > 0 \rightarrow m > -4 \\ \textcircled{1} \quad \textcircled{2} \quad \textcircled{3} &\rightarrow -4 < m \leq -f \end{aligned}$$

$$\begin{aligned} rx^2 + (4m-1)x + r-m &\geq 0 \quad \text{substituting } f \\ f(m+1) - f(m) - f(4-m) &\geq 0 \rightarrow f(m+1) - f(m) - f(4-m) \geq 0 \\ -1 \pm \sqrt{4f - f(-1)} &\quad -1 \pm \sqrt{r^2(r+4)} \quad -r \pm \sqrt{r^2} \quad \frac{-1 \pm \sqrt{r^2}}{r} \quad \frac{-1 \pm \sqrt{r^2}}{r} \end{aligned}$$

$$\begin{aligned} \textcircled{1} m \in (-\infty, -1 - \frac{\sqrt{r^2}}{r}) \cup (-1 + \frac{\sqrt{r^2}}{r}, +\infty) \quad \textcircled{2} \alpha + \beta = \frac{1}{\alpha\beta} \quad \frac{-r+1}{r} &= \frac{1}{\frac{r-m}{r}} \end{aligned}$$

$$\begin{aligned} rm^2 - 4m - 1 &\geq 0 \quad (rm-1)(m+1) = 0 \rightarrow m = \begin{cases} \textcircled{1} -1/r \\ \textcircled{2} -1 \end{cases} \quad \textcircled{1} \textcircled{2} \text{ if } m = \begin{cases} -1/r \\ -1 \end{cases} \end{aligned}$$

$$x^2 - x - f \geq 0 \quad \delta \geq 1 \quad p = -f \quad \frac{x^2}{r} + \frac{x^2}{r} + \frac{1}{x} + \frac{1}{x} =$$

$$\begin{aligned} &= (x_1 + x_2) - r(r)(x_1 + x_2) + \frac{x_1 + x_2}{r} \geq 1 + 1 + \frac{1}{r} = \frac{2r+1}{r} \\ (\frac{x^2}{r} + \frac{1}{x_1})(\frac{x^2}{r} + \frac{1}{x_2}) &= (\frac{x_1 x_2}{r}) + \frac{x_1^2}{r} + \frac{x_2^2}{r} + \frac{1}{x_1 x_2} = 4f + 9 - \frac{1}{r} = -\frac{r-1}{r} \end{aligned}$$

$$y = \frac{x^2}{r} - \frac{2r-1}{r}x - \frac{r-1}{r}$$

$$\sqrt{x^2} + (t + \frac{1}{t} + 1)(t-1) \cdot \frac{t^2-1}{t} \rightarrow \frac{r\sqrt{x^2}-1}{r\sqrt{x^2}} \quad r\sqrt{x^2}$$

$$\frac{r\sqrt{x^2}-1}{r\sqrt{x^2}} \quad r\sqrt{x^2} \quad x^2 - 1 \geq 2rx \quad x^2 - 2rx - 1 \geq 0 \quad \delta \geq r$$

1 /

$$rx - ax + f = 0 \quad \alpha \neq \beta \rightarrow P = \frac{f}{r} \quad r\beta = \frac{f}{r} \quad \beta = \frac{f}{r} \quad 9$$

$$\beta \rightarrow \frac{r}{r} \quad L \rightarrow -\frac{r}{r} \quad \begin{array}{c} r \\ \swarrow \\ -r \end{array} \quad \delta \begin{array}{c} \frac{1}{r} = \frac{\alpha}{r} \quad \alpha = 1 \\ -\frac{1}{r} = \frac{\alpha}{r} \rightarrow \alpha = -1 \end{array} \quad \Delta + \Delta = 14$$

$$y = ax^2 + (r+ra)x \rightarrow \textcircled{1} a > 0 \quad \textcircled{2} \quad \begin{array}{c} \text{Graph of a parabola opening upwards} \end{array} \quad 14$$

$$\alpha \quad \text{در این صورت که} \quad \begin{array}{c} \text{Graph of a parabola opening upwards} \end{array} \quad \Delta \geq 0 \quad 9 + f^2 + 12a \geq 0 \quad (ra+r)^2 \geq 0 \quad \textcircled{5} a \neq -\frac{r}{r}$$

$$\textcircled{6} \delta \geq 0 \rightarrow -\frac{r-r_0}{a} \geq 0 \quad -\frac{r}{r} > 0 \quad \textcircled{1} \textcircled{2} \textcircled{3} \textcircled{4} \textcircled{5} \quad \text{در این صورت که}$$

$$y = -x^2 - rx + b \quad y = x^2 + ax - r \quad \frac{-b}{10} = \frac{r}{-r} = \frac{-a}{r} \quad a \geq r \quad 14$$

$$1 = x^2 - rx + b \quad r = -r + b \quad b \geq r \quad ab = rxr = 1$$

$$rx^2 + ax - 4 \geq 0 \quad \delta = \frac{-a}{ra} = \frac{-1}{r} \quad P = \frac{-4}{r} = -\frac{4}{r} \quad rx^2 - 4x + b \geq 0$$

$$\delta = \alpha + \frac{1}{r} + \beta + \frac{1}{r} = \frac{-1}{r} + \frac{1}{r} + \frac{1}{r} + \frac{1}{r} = \frac{-(-a)}{r} \quad \alpha = 1$$

$$P = (\alpha + \frac{1}{r})(\beta + \frac{1}{r}) = a\beta + \frac{1}{r}(\alpha + \beta) + \frac{1}{r} = \frac{b}{r} = -\frac{r}{r} = \frac{-1}{r} + \frac{1}{r} = \frac{b}{r}$$

$$\boxed{b = -4} \quad ab = (-4) \times 1 = -4 \quad \left[\frac{ab}{r} \right] = \left[\frac{-4}{r} \right] = -\frac{4}{r}$$

$$x^2 + 4x + m \geq 0 \quad \delta = -4 \quad \alpha = -4 - \beta \quad x^2 + rx - rm \geq 0 \quad \delta = -r$$

$$\alpha = -r - \delta \quad \textcircled{1}, \textcircled{2} \quad -4 - \beta = -r - \delta \quad \delta - \beta = \textcircled{5}$$