۱) پریسا قراخانی تکلیف ۲۳ دهم (قرآن)

$\frac{-b}{2a}=-1 \to b=2a$, $x=-1 \to a-b+c=9 \to -a+c=9$

$x=3 \to 9a+3b+c=1 \to 16a=-8 \to a=-\frac{1}{2} \to b=2(-\frac{1}{2})=0$

$c=9-\frac{1}{2}=\frac{17}{2}$

$$\boxed{y=-\frac{1}{2}x^2-x+\frac{17}{2}}$$

۲)

$y=2x^2+mx+m+6=0 \to$ دو ریشه دارد $\Rightarrow m^2-4(2m+12)>0$

$\to m^2-8m-48>0 \to (m+4)(m-12)>0 \to$

x		-4		12	
y	$+$		$-$		$+$

$\Rightarrow$ ① $m\in(-\infty,-4)\cup(12,+\infty)$

② $S=-\frac{m}{2}>0 \to m<0$

③ $\frac{m+6}{2}>0 \to m>-6$

$\xrightarrow{\cap} -6<m<-4$

۳)

$3x^2+(2m-1)x+2-m=0 \to$ دو ریشه دارد $\Rightarrow 4m^2+1-4m-4(6-3m)>0$

$\Rightarrow 4m^2+8m-23>0 \to m=\frac{-8\pm\sqrt{64-4(-92)}}{8}=\frac{-8\pm\sqrt{2^4(27)}}{8}$

$=\frac{-2\pm\sqrt{27}}{2} \Rightarrow$

x		$-1-\frac{\sqrt{27}}{2}$		$-1+\frac{\sqrt{27}}{2}$	
y	$+$		$-$		$+$

$\Rightarrow$

① $m\in(-\infty,-1-\frac{\sqrt{27}}{2})\cup(-1+\frac{\sqrt{27}}{2},+\infty)$

② $\alpha+\beta=\frac{1}{\alpha\beta} \Rightarrow \frac{-2m+1}{3}=\frac{1}{\frac{2-m}{3}} \to 2m^2-5m-7=0$

$(2m-7)(m+1)=0$

$\Rightarrow m \begin{cases} ① & -1 \ ✗ \\ ② & \frac{7}{2} \end{cases} \xrightarrow{\cap} \boxed{m=\frac{7}{2}}$

۴)

$x^2 - x - 4 = 0 \to S = 1,\ P = -4$

$x_2^3 + x_1^3 + \frac{1}{x_2} + \frac{1}{x_1} = (x_1+x_2)^3 - 3(-4)(x_1+x_2) + \frac{x_1+x_2}{x_1 x_2}$ ($x_1x_2 = -4$)

$\Rightarrow 1 + 12 + \frac{-1}{4} = \frac{51}{4} = S$

$(x_2^3 + \frac{1}{x_1})(x_1^3 + \frac{1}{x_2}) = (x_1 x_2)^3 + \underbrace{x_1^2 + x_2^2}_{1-2(-4)=9} + \frac{1}{x_1 x_2} = -64 + 9 - \frac{1}{4} = -\frac{221}{4} = P$

$\boxed{y = x^2 - \frac{51}{4}x - \frac{221}{4}}$

۵)

$\sqrt[3]{x^2} = t \to (t + \frac{1}{t} + 1)(t-1) = t^2 - \frac{1}{t} \to \sqrt[3]{x^4} - \frac{1}{\sqrt[3]{x^2}} = 2\sqrt[3]{x}$

$\Rightarrow \frac{\sqrt[3]{x^4 - 1}}{\sqrt[3]{x^2}} = 2\sqrt[3]{x} \to x^2 - 1 = 2x \to x^2 - 2x - 1 = 0$

$-S = -2 \qquad S = 2$

$\boxed{S = 2}$

۶)

$3x^2 - \alpha x + 4 = 0 \quad \alpha = 3\beta \to P = \frac{4}{3} \Rightarrow 3\beta^2 = \frac{4}{3} \to \beta^2 = \frac{4}{9}$

$\to \alpha$ ① 2, ② -2 $\Rightarrow S$: ① $\frac{8}{3} = \frac{\alpha}{3}$, $\alpha = 8$; ② $\frac{-8}{3} = \frac{\alpha}{3}$, $\alpha = -8$

β: ① $\frac{2}{3}$ ② $-\frac{2}{3}$

$8 - (-8) = 16$

۱۰)

1) $x^2 + 6x + m = 0 \to S = -6 \to \alpha = -6 - \beta$

ریشه مشترک

2) $x^2 + 2x - 3m = 0 \to S = -2 \to \alpha = -2 - \gamma$

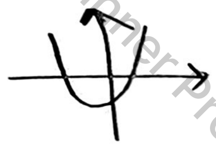
$-6 - \beta = -2 - \gamma$

$\boxed{\gamma - \beta = 4}$

(۷)

$y = ax^2 + (3+2a)x \Rightarrow$

① $a > 0$

② 

$\Delta \geq 0 \rightarrow 9 + 4a^2 + 12a \geq 0$

$(2a+3)^2 \geq 0 \rightarrow a \neq -\frac{3}{2}$

③ $S \geq 0 \quad \frac{-3-2a}{a} \geq 0 \rightarrow -\frac{3}{2} > a$

$\xrightarrow{\cap} \varnothing$ هیچ مقدار

(۸)

$y = -x^2 - 2x + b \ , \ y = x^2 + ax - 2$

با $\frac{-b}{2a} = \frac{2}{-2} = \frac{-\alpha}{2} \rightarrow \alpha = 2$

$\begin{cases} 1 = -x^2 - 2x + b \\ 1 = x^2 + 2x - 2 \end{cases} \rightarrow 2 = -2 + b \rightarrow b = 4 \qquad a \times b = 2 \times 4 = 8$

(۹)

$2x^2 - ax + b = 0 \rightarrow S = \alpha + \frac{1}{2} + \beta + \frac{1}{2} = -\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$

$= \frac{-(-a)}{2} \rightarrow \alpha = 1$

$P = (\alpha + \frac{1}{2})(\beta + \frac{1}{2}) = \alpha\beta + \frac{1}{2}(\alpha + \beta) + \frac{1}{4} = \frac{+b}{2}$

$\rightarrow -3 + \frac{1}{2}(-\frac{1}{2}) + \frac{1}{4} = \frac{+b}{2} \rightarrow b = -6$

$ab = (-6) \times 1 = -6 \rightarrow \left[\frac{ab}{4}\right] = \left[-\frac{3}{2}\right] = -2$