نام و نام خانوادگی ...آیدا [illegible]............ پاسخنامه تشریحی تکلیف شماره ...۲۳... کلاس ....

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$y = a(x - x_s)^2 + y_s$

$y = a(x+1)^2 + 9 \xrightarrow{(3,1)} 1 = a(4)^2 + 9 \qquad 16a = -8 \to a = -\frac{1}{2}$

$\Rightarrow y = -\frac{1}{2}(x+1)^2 + 9 \longrightarrow y = -\frac{1}{2}(x^2 + 1 + 2x) + 9 \to y = -\frac{1}{2}x^2 - x \cancel{-\frac{1}{2} + 9\frac{18}{2}}$

$+\frac{17}{2}$

$\boxed{y = \frac{-1}{2}x^2 - x + \frac{17}{2}}$

۲

$\Delta > 0 \qquad m^2 - 8(m+4) > 0 \qquad m^2 - 8m - 48 > 0 \qquad (m-12)(m+4) > 0$

$\hookrightarrow 12 \qquad \hookrightarrow -4$

$\begin{array}{ccccc} & -4 & & 12 & \\ + & 0 & - & 0 & + \end{array}$

$(-\infty, -4) \cup (12, +\infty)$

$\frac{-b}{a} > 0 \qquad \frac{-m}{2} > 0 \qquad m < 0$

$\frac{c}{a} > 0 \qquad \frac{m+4}{2} > 0 \qquad m + 4 > 0 \qquad m > -4$

$\xRightarrow{\cap} \boxed{(-4, -4)}$

۳

$\frac{-b}{a} = \frac{a}{c} \Rightarrow a^2 = -bc$

$9 = -(2m-1)(2-m) \to -(\overset{5m}{\cancel{4m}} - 2m^2 - 2 + \cancel{m}) \Rightarrow 2m^2 - 5m + \cancel{2} = \cancel{9}$

$2m^2 - 5m - 7 = 0$

$m^2 - 5m - 14 = 0$

$(m-7)(m+2)$

$7 \quad -2 \xrightarrow{\div 2} \frac{7}{2}, -1$

$\Delta > 0 \quad (2m-1)^2 - 12(2-m) > 0$

$4m^2 - \cancel{4m} + 1 \cancel{-24} + 12m > 0$ (-23, $8m$)

$4m^2 + 8m - 23 > 0 \qquad m = \frac{-8 \pm \sqrt{432}}{8} < \begin{matrix} m_1 \to m_1 < -1 \\ m_2 \end{matrix} \qquad \begin{array}{ccccc} & m_1 & & m_2 & \\ + & | & - & | & + \end{array}$

$\xrightarrow{\cap} \boxed{m = \frac{7}{2} = 3{,}5}$

۴

$x^2 - x - 4 = 0 < \begin{matrix} \alpha \\ \beta \end{matrix} \qquad \alpha^3 + \frac{1}{\beta}, \ \beta^3 + \frac{1}{\alpha}$

$\to \frac{c}{a} = (\alpha^3 + \frac{1}{\beta})(\beta^3 + \frac{1}{\alpha}) = \alpha^3\beta^3 + \alpha^2 + \beta^2 + \frac{1}{\alpha\beta}$

$\to \frac{-b}{a} = \alpha^3 + \beta^3 + \frac{1}{\alpha} + \frac{1}{\beta} = \alpha^3 + \beta^3 + \frac{\alpha + \beta}{\alpha\beta}$

$\alpha^3\beta^3 = S^3 - 3SP \Rightarrow 1 + 3 \times 4 = 13$

$\alpha^2\beta^2 = S^2 - 2P \Rightarrow 1 + 8 = 9$

$\to S = \frac{-b}{a} = 1$

$P = \frac{c}{a} = -4$

$\to \frac{c}{a} = \cancel{-64} + 9 + \left(\frac{-1}{4}\right) = \frac{-221}{4}$ (-55)

$\frac{-b}{a} = 13 + \left(\frac{-1}{4}\right) = \frac{51}{4}$

$x^2 - Sx + P = 0$

$x^2 - \frac{51}{4} - \frac{221}{4} = 0 \Rightarrow$

$\boxed{4x^2 - 51 - 221 = 0}$

۵

$t = \sqrt[3]{x} \qquad \Rightarrow (t^2 + 1 + \frac{1}{t})(t^2 - 1) = 2t \Rightarrow t^4 - 1 + t - \frac{1}{t} = 2t$

$(t^4 - 1 - t - \frac{1}{t} = 0)t \qquad t^5 - t^2 - t - 1 = 0 \xrightarrow[t=-1]{t=1} \begin{matrix} 1 - 1 - 1 - 1 \neq -2 & ✗ \\ -1 - 1 + 1 - 1 = -2 & ✗ \end{matrix}$

$t \neq 0$

$t^4 - 1 - t + \frac{1}{t} = 0 \qquad (t^4 - 1) - (t + \frac{1}{t}) = 0 \qquad t^4 - 1 = (t^2 - 1)(t^2 + 1) \Rightarrow$

$(t-1)(t^4 + t^3 + t^2 - 1) = 0$

$t = 1, \ t = -1 \qquad t = \sqrt[3]{x} < \begin{matrix} x = 1 \\ x = -1 \end{matrix} \Big\} \ 1 + (-1) = 0$

۶	$\begin{cases}\alpha\\ \beta\end{cases} \to \beta=۳\alpha$ $\frac{-b}{a}=۳\alpha+\alpha=۴\alpha$ $\frac{a}{۳}=۴\alpha$ $\frac{c}{a}=۳\alpha\times\alpha=۳\alpha^۲=\frac{۴}{۳}$ $\alpha^۲=\frac{۴}{۹}$ $\alpha=\pm\frac{۲}{۳}$ $\frac{a}{۳}=۴\alpha$ ⟨ $\alpha=\frac{۲}{۳}$ $\frac{a}{۳}=\frac{۸}{۳}\to a_۱=۸$; $\alpha=\frac{-۲}{۳}$ $\frac{a}{۳}=\frac{-۸}{۳}\to a_۲=-۸$ ⟩ $\lvert a_۲-a_۱\rvert=\lvert ۸-(-۸)\rvert=۱۶$
۷	$ax^۲+(۳+۲a)x$ $a>۰$ ✓ $\frac{c}{a}=۰$ ✓ $\frac{-b}{a}\geq۰$ $\frac{-(۳+۲a)}{a}\geq۰$ $\frac{-۳}{۲}$ $\begin{array}{c\|c\|c} \frac{-۳}{۲} & & ۰\\ - & + & - \end{array}$ $\to[-\frac{۳}{۲},۰)$ $\xrightarrow{\cap}$ $\emptyset$ بازای هیچ مقدار a $\Delta\geq۰$ $\frac{(۳+۲a)^۲}{۲a}\geq۰$ $\frac{-۳}{۲}$ ، $۲a$ ↳ $\frac{-۱}{۲}$ $\begin{array}{c\|c\|c} \frac{-۳}{۲} & & \frac{-۱}{۲}\\ - & - & + \end{array}$ $\to(\frac{-۱}{۲},+\infty)$
۸	$x=\frac{-b}{a}$ $\frac{-a}{۱}=\frac{۲}{-۱}\Rightarrow a=۲$ $y=x^۲+۲x-۲$ $\to$ $۱=x^۲+۲x+b$ $x^۲+۲x+(b-۱)=۰$ $y=-x^۲-۲x+b$ $۱=-x^۲-۲x+b$ $x^۲+۲x+(۱-b)=۰$ $\to$ $۱-b=b-۱$ $۲b=۲$ $b=۱$ $ab=\boxed{۲}$
۹	$۲x^۲-ax+b$ $\begin{cases}\alpha+۰/۵\\ \beta+۰/۵\end{cases}$ $\frac{-b}{a}=\alpha+\beta+۱=\frac{a}{۲}=\frac{-۱}{۲}+۱=\frac{۱}{۲}\Rightarrow a=۱$ $\frac{c}{a}=۰/۲۵+(\alpha+\beta)۰/۵+\alpha\beta=\frac{b}{۲}=\cancel{\frac{۱}{۴}}+\overset{\frac{-۱}{۴}}{\cancel{(-\frac{۱}{۲})(\frac{۱}{۲})}}+\frac{-۶}{\cancel{۲a}}\Rightarrow b=-۶$ $۲ax^۲+ax-۶$ $\begin{cases}\alpha\\ \beta\end{cases}$ $\frac{-b}{a}=\alpha+\beta=\frac{-\cancel{a}}{۲\cancel{a}}=\frac{-۱}{۲}$ $\frac{c}{a}=\alpha\beta=\frac{-۶}{۲a}$ $\left[\frac{-۶\times۱}{۴}\right]=-۲$
۱۰	$(x^۲+۶x+m)-(x^۲+۲x-۳m)=۰$ $۴x+۴m=۰$ $x+m=۰$ برای ریشه مشترک $x=-m$ $\to(-m^۲)+۶(-m)+m=۰$ $m^۲-۶m+m=۰$ $m^۲-۵m=۰$ $m(m-۵)=۰$ $\to m=۰$ غ ق ق ، $\to ۵$ $m=۵$ $\xrightarrow{\text{ریشه مشترک}}$ $x=-۵$ $x^۲+۶x+۵=۰$ $(x+۱)(x+۵)=۰$ $\to -۱$ ، $\to -۵$ $x^۲+۲x-۱۵=۰$ $(x+۵)(x-۳)=۰$ $\to -۵$ ، $\to ۳$ $\boxed{۳-(-۱)=۴}$