نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره ۲.۳... کلاس ......

۱

$S(h,k) \to y=a(x-h)^2+k$

$1=a(3+1)^2+9 \Rightarrow -8=16a \Rightarrow a=-\frac{1}{2}$

$\boxed{y=-\frac{1}{2}(x+1)^2+9}$

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$2x^2+mx+(m+6)=0$

$\Delta>0 \to \Delta=b^2-4ac=m^2-4(2)(m+6)=m^2-8m-48>0 \to m<-4 \;,\; m>12$

$\boxed{m\in(-\infty,-4)\cup(12,+\infty)}$

$S>0 \to S=-\frac{b}{a} \to S=-\frac{m}{2}>0 \to \boxed{m<0}$

$P>0 \to P=\frac{c}{a} \quad P=\frac{m+6}{2}>0 \to m+6>0 \to \boxed{m>-6}$

اشتراک: $\boxed{-6<m<-4}$

۳

$S=\frac{1}{P}$

$3x^2+(2m-1)x+(2-m)=0 \Rightarrow \begin{cases} S=-\frac{b}{a}=\frac{2m-1}{3} \\ P=\frac{c}{a}=\frac{2-m}{3} \end{cases}$

$S=\frac{1}{P} \longrightarrow -\frac{2m-1}{3}=\frac{3}{2-m} \Rightarrow (2m-1)(m-2)=9 \Rightarrow 2m^2-4m-m+2=9$

$2m^2-5m-7=0 \to (2m-7)(m+1)=0 \to \cancel{m=-1} \;,\; \boxed{m=\frac{7}{2}}$

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$x=x^2-4 \Rightarrow x^2-x-4=0 \Rightarrow \begin{cases} S=x_1+x_2=\frac{-b}{a}=1 \\ P=x_1x_2=\frac{c}{a}=-4 \end{cases}$

$S'=(x_1^3+\frac{1}{x_2})+(x_2^3+\frac{1}{x_1})=(x_1^3+x_2^3)+\left(\frac{x_1+x_2}{x_1x_2}\right)=S^3-3PS+\frac{S}{P}=1+12-\frac{1}{4}=\frac{51}{4}$

$P'=(x_1^3+\frac{1}{x_2})(x_2^3+\frac{1}{x_1})=x_1^3x_2^3+x_1^2+x_2^2+\frac{1}{x_1x_2}=P^3+S^2-2P+\frac{1}{P}=-64+1+8-\frac{1}{4}=\frac{-221}{4}$

$x^2-S'x+P'=0 \longrightarrow x^2-\frac{51}{4}x-\frac{221}{4}=0 \xrightarrow{\times 4} 4x^2-51x-221=0$

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$\left(\sqrt[3]{x^2}+\frac{1}{\sqrt[3]{x^2}}+1\right)\left(\sqrt[3]{x^2}-1\right)=2\sqrt[3]{x} \Rightarrow \left(\frac{\sqrt[3]{x^2}\times\sqrt[3]{x^2}+1+\sqrt[3]{x^2}}{\sqrt[3]{x^2}}\right)\left(\sqrt[3]{x^2}-1\right)=2\sqrt[3]{x}$

$\Rightarrow \frac{(\sqrt[3]{x^4}+\sqrt[3]{x^2}+1)(\sqrt[3]{x^2}-1)}{\sqrt[3]{x^2}}=2\sqrt[3]{x} \Rightarrow \frac{(\sqrt[3]{x^2})^3-1^3}{\sqrt[3]{x^2}}=2\sqrt[3]{x}$

$\Rightarrow x^2-1=(2\sqrt[3]{x})(\sqrt[3]{x^2}) \Rightarrow x^2-1=2x \Rightarrow x^2-2x-1=0 \Rightarrow S=\frac{-b}{a}=\frac{-(-2)}{1}=2$

۶

$3x^2 - ax + 4 = 0 \to \alpha, \beta$

$\alpha = 3\beta \xrightarrow{\times\alpha} \alpha^2 = 3\alpha\beta \xrightarrow{P=\alpha\beta=\frac{4}{3}} \alpha^2 = 3\left(\frac{4}{3}\right) \Rightarrow \alpha^2 = 4 \Rightarrow \alpha = \pm 2$

$3x^2 - ax + 4 = 0 \xrightarrow{\alpha=2} 3\times 4 - 2a + 4 = 0 \to a = 8$

$3x^2 - ax + 4 = 0 \xrightarrow{\alpha=-2} 3\times 4 + 2a + 4 = 0 \to a = -8$

$8 - (-8) = 16$

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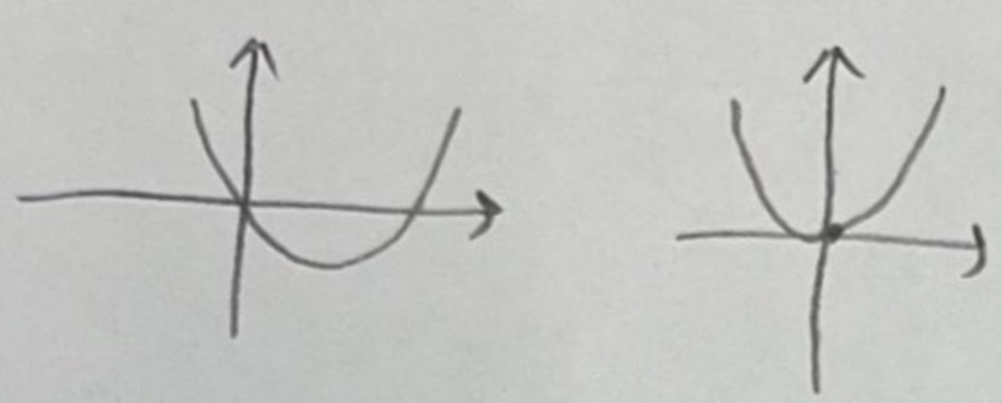

$a > 0$

نقطه تقاطع با xها ≥ 0 ★

تقاطع با محور xها $\to y = 0 \Rightarrow x(ax + 3 + 2a) = 0 \Rightarrow \begin{cases} x = 0 \\ ax + 3 + 2a \end{cases}$

$x = -\frac{3+2a}{a}$

★ $\to -\frac{3+2a}{a} \geq 0 \Rightarrow \frac{3+2a}{a} \leq 0$

$\xrightarrow{a>0} 3 + 2a \leq 0 \to a \leq -\frac{3}{2} \xrightarrow{a>0}$ اشتراک وجود ندارد

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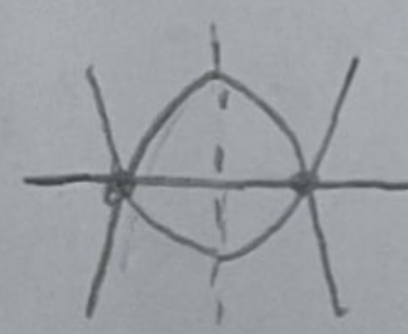
$y = x^2 + ax - 2 \longrightarrow$ محور تقارن: $x = -\frac{a}{2}$

$y = -x^2 - 2x + b \longrightarrow$ محور تقارن: $x = -\frac{-2}{2(-1)} = -1 \longrightarrow -\frac{a}{2} = -1 \to a = 2$



$y = 1$

$x = -1$

$x^2 + 2x - 2 = 1 \to x^2 + 2x - 3 = 0 \to (x+3)(x-1) = 0$

$x = -3$, $x = 1$

$(-3,1)$ و $(1,1)$

$y = -x^2 - 2x + b \xrightarrow{(1,1)} 1 = -1 - 2 + b \to b = 4$ $\quad ab = 2\times 4 = 8$

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$2ax^2 + ax - 6 = 0 \to \alpha, \beta$ $\qquad 2x^2 - ax + b = 0 \to \beta + \frac{1}{2},\ \alpha + \frac{1}{2}$

$S = \alpha + \beta = -\frac{a}{2a} = -\frac{1}{2}$, $P = \alpha\beta = -\frac{6}{2a} = -\frac{3}{a}$

$S' = -\frac{-a}{2} \Rightarrow (\alpha + \frac{1}{2}) + (\beta + \frac{1}{2}) = \frac{a}{2} \to \alpha + \beta + 1 = \frac{a}{2} \to -\frac{1}{2} + 1 = \frac{a}{2} \to a = 1$

$P' = \frac{b}{2} \Rightarrow (\alpha + \frac{1}{2})(\beta + \frac{1}{2}) = \frac{b}{2} \Rightarrow \alpha\beta + \frac{1}{2}(\alpha+\beta) + \frac{1}{4} = \frac{b}{2} \Rightarrow -\frac{3}{1} + \frac{1}{2}(-\frac{1}{2}) + \frac{1}{4} = \frac{b}{2} \to b = -6$

$\left[\frac{ab}{2}\right] = \left[\frac{(1)(-6)}{2}\right] = \left[-\frac{3}{2}\right] = [-1/5] = -2$

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$\begin{cases} x^2 + 6x + m = 0 \Rightarrow \text{مجموع ریشه‌ها}: \alpha + \beta_1 = -6 \\ x^2 + 2x - 3m = 0 \Rightarrow \text{مجموع ریشه‌ها}: \alpha + \beta_2 = -2 \end{cases}$

$\xrightarrow{\text{تفاضل طرفین}} (\alpha + \beta_1) - (\alpha + \beta_2) = -6 - (-2)$

$\Rightarrow \beta_1 - \beta_2 = -4$

$|\beta_1 - \beta_2| = 4$