

سوال ۱) فرض: $\frac{-b}{2a} = -1 \rightarrow b = 2a$ $\frac{-\Delta}{4a} = \frac{-b^2 + 4ac}{4a} = \frac{-4a^2 + 4ac}{4a} = \frac{4a(c-a)}{4a} = c-a = 9$
 $c = a+9$
 $y = ax^2 + 2ax + a+9 \xrightarrow{(3,1)} 1 = 9a + 4a + a + 9 \rightarrow 14a = -8 \rightarrow a = -\frac{1}{2}$
 $y = -\frac{1}{2}x^2 - x + \frac{17}{2}$

سوال ۲) $\Delta > 0 \rightarrow b^2 - 4ac > 0 \rightarrow m^2 - 4(2)(m+4) > 0$ $m^2 - 8m - 32 > 0$
 $(m-12)(m+4) > 0$ $\begin{array}{c} -4 \quad 12 \\ + \quad | \quad - \quad | \quad + \end{array}$
 $S > 0 \rightarrow \frac{-b}{a} > 0 \rightarrow \frac{-m}{2} > 0 \rightarrow m < 0$
 $P > 0 \rightarrow \frac{c}{a} > 0 \rightarrow \frac{m+4}{2} > 0 \rightarrow m > -4$ $\text{انترال هم } (-4, -2) \rightarrow \text{جواب}$

سوال ۳) $S = \frac{-b}{a}$ $P = \frac{c}{a} \xrightarrow{\text{عکس}} \frac{a}{c}$ $\frac{-b}{a} = \frac{a}{c} \rightarrow a^2 = -bc$
 $\Delta > 0 \rightarrow b^2 - 4ac > 0$ $9 = -(2m-1)(2-m) = 2m^2 - 4m + 2$
 $m = -1 \rightarrow (2(-1)-1)^2 - 4(2)(2-(-1)) = -27 < 0$ غ ق ق
 $m = \frac{5}{2} \rightarrow (2(\frac{5}{2})-1)^2 - 4(2)(2-\frac{5}{2}) = 45 > 0$ $\checkmark \rightarrow \frac{5}{2}$ قبل قبل $\frac{5}{2}$ پس $\frac{5}{2}, \frac{2}{5}$

سوال ۴) $x^2 - 5x + P = 0$ $S = x_1^3 + x_2^3 + \frac{1}{x_1} + \frac{1}{x_2} = (x_1 + x_2)^3 - 3x_1x_2(x_1 + x_2) + \frac{x_1 + x_2}{x_1x_2}$
 $P = (x_1^3 + \frac{1}{x_1})(x_2^3 + \frac{1}{x_2}) = (x_1x_2)^3 + x_1^2 + x_2^2 + \frac{1}{x_1x_2} = (x_1x_2)^3 + (x_1 + x_2)^2 - 2x_1x_2 + \frac{1}{x_1x_2}$
 $x^2 - 5x - 4 = 0 \rightarrow x_1x_2 = \frac{c}{a} = -4$ $x_1 + x_2 = \frac{-b}{a} = 5$
 $S = 1^3 - 3(-4)(1) + \frac{1}{-4} = \frac{41}{4}$ $P = (-4)^3 + 1^2 - 2(-4) + \frac{1}{-4} = \frac{-221}{4}$ $\boxed{x^2 - \frac{41}{4}x - \frac{221}{4} = 0}$

سوال ۵) $\sqrt{x^4} - \sqrt{x^2} + 1 - \frac{1}{\sqrt{x^2}} + \sqrt{x^2} - 1 = \sqrt{x^4} - \frac{1}{\sqrt{x^2}} = \frac{\sqrt{x^4} - 1}{\sqrt{x^2}} \rightarrow \frac{x^2 - 1}{\sqrt{x^2}} = 2\sqrt{x}$
 $x^2 - 1 = 2x \rightarrow x^2 - 2x - 1 = 0 \rightarrow S = \frac{-b}{a} \rightarrow \frac{-(-2)}{1} = 2$

سوال ۶) فرض: $\alpha, 3\alpha$ $P = 3\alpha^2$ $S = 4\alpha$
 $P = \frac{c}{a} = \frac{4}{3} = 3\alpha^2 \rightarrow \alpha^2 = \frac{4}{9} \rightarrow \alpha = \pm \frac{2}{3}$ $S = \frac{-b}{a} = \frac{+a}{3} = 4\alpha \rightarrow \begin{array}{l} \alpha = \frac{2}{3} \quad a = 1 \\ \alpha = -\frac{2}{3} \quad a = -1 \end{array}$
 اختلاف: $1 - (-1) = 2$

$$a > 0 \quad \Delta = b^2 - 4ac = (r+2a)^2 \geq 0 \rightarrow \frac{-b}{a} \geq 0 \quad \frac{-(r+2a)}{a} \geq 0 \quad \frac{-\frac{r}{a} - 2}{1 + \frac{2}{a}} \geq 0 \quad (\text{سوال ۷})$$

$$(0, +\infty) \cap \left[-\frac{r}{a}, 0\right) = \emptyset$$

$$\text{نقطهٔ بحرانی: } x = -\frac{b}{2a} \rightarrow \frac{-a}{r} = \frac{-(-r)}{-r} \rightarrow a = r \quad (\text{سوال ۸})$$

$$y = 1 \rightarrow \begin{cases} 1 = x^2 + 2x - 2 \\ x^2 + 2x - 3 = 0 \end{cases} \quad \begin{cases} 1 = -x^2 - 2x + b \\ x^2 + 2x - b + 1 = 0 \end{cases} \quad \left\{ \begin{array}{l} x^2 + 2x - 3 = x^2 + 2x - b + 1 \\ -3 = -b + 1 \rightarrow b = 4 \end{array} \right. \quad ab = 2 \times 4 = 8$$

$$2ax^2 + ax - 4 = 0 \rightarrow S_1 = \frac{-a}{2a} = -\frac{1}{2} \quad 2x^2 - ax + b = 0 \rightarrow S_2 = \frac{a}{2} = S_1 + \frac{1}{2} + \frac{1}{2} \rightarrow a = 1 \quad (\text{سوال ۹})$$

(ریشه: α, β)

$$P_1 = \frac{-4}{2 \times 1} = -2 \quad P_2 = \left(\alpha + \frac{1}{2}\right)\left(\beta + \frac{1}{2}\right) = \alpha\beta + \frac{\alpha+\beta}{2} + \frac{1}{4} = (-3) + \frac{-\frac{1}{2}}{2} + \frac{1}{4} = -2 \quad P_2 = \frac{b}{2} = -2 \quad b = -4$$

$$\left[\frac{ab}{4}\right] = \left[\frac{1 \times (-4)}{4}\right] = -1$$

$$\text{ریشه مشترک: } \alpha \quad \begin{cases} \alpha^2 + 4\alpha + m = 0 \\ \alpha^2 + 2\alpha - 3m = 0 \end{cases} \rightarrow \begin{cases} \alpha^2 + 4\alpha + m = \alpha^2 + 2\alpha - 3m \\ 2\alpha = -4m \\ \alpha = -2m \end{cases} \quad (\text{سوال ۱۰})$$

$$\checkmark \text{ جایگزینی: } (-m)^2 + 4(-m) + m = 0 \rightarrow m^2 - 3m = 0 \rightarrow m(m-3) = 0 \quad \left\{ \begin{array}{l} m = 0 \\ m = 3 \end{array} \right. \quad \text{طبق سوال غنای}$$

$$\begin{aligned} x^2 + 4x + \omega &= 0 & x^2 + 2x - 1\omega &= 0 \\ (x+1)(x+\omega) &= 0 \rightarrow \begin{cases} x = -1 \\ x = -\omega \end{cases} & (x-3)(x+\omega) &= 0 \rightarrow \begin{cases} x = 3 \\ x = -\omega \end{cases} \end{aligned}$$

اطلاعات ریشه: $3 - (-1) = 4$
غیر مشترک