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آیتین جلسه پاسخ تکلیف ۲۱ کلاس ۶

(۱) $\frac{(۱۱!+۳۱+\underbrace{۵!+۱۰۰+۲۵!}_{\text{یکان صفر}})(۳!+۴!+\underbrace{۶!+۱۰۰+۲۵!}_{\text{یکان صفر}})}{\underbrace{۱+۶=۷}\quad\quad\underbrace{۲+۴=۶}}$

$\downarrow$

$۷\times۶=۴۲$ (۲)

الف)

$\sqrt[۴]{(۴+\sqrt{۷})^{-۱}}\times\sqrt{۱+\sqrt{۷}}=\sqrt[۴]{(۴+\sqrt{۷})^{-۱}}\times\sqrt[۲\times۲]{(۱+\sqrt{۷})^{۲}}$ (۲)

$=\sqrt[۴]{(۴+\sqrt{۷})^{-۱}}\times\sqrt[۴]{۱+۷+۲\sqrt{۷}}=\sqrt[۴]{(۴+\sqrt{۷})^{-۱}(۸+۲\sqrt{۷})}=\sqrt[۴]{\frac{۱}{۴+\sqrt{۷}}\times۲(۴+\sqrt{۷})}=\sqrt[۴]{۲}$

ب) $A=\sqrt{۳-\sqrt{۵}}-\sqrt{۳+\sqrt{۵}}$

$A^۲=(۳-\sqrt{۵})+(۳+\sqrt{۵})-۲\sqrt{۳-\sqrt{۵}}\times\sqrt{۳+\sqrt{۵}}=۶-۲\sqrt{(۳-\sqrt{۵})(۳+\sqrt{۵})}$

$=۶-۲\sqrt{۴}=۶-۴=۲\longrightarrow A^۲=۲\ \ A<۰\ \ A=-\sqrt{۲}$

$\left(\frac{\sqrt{۲}+\sqrt{۵}}{\sqrt{۱۰}+۲}\right)(-\sqrt{۲})=\frac{-(\sqrt{۴}+\sqrt{۱۰})}{\sqrt{۱۰}+۲}=\frac{-(۲+\sqrt{۱۰})}{\sqrt{۱۰}+۲}=-۱$

الف) $A=\frac{\sqrt{۸}+\sqrt{۲۷}}{۵-\sqrt{۶}}=\frac{۲\sqrt{۲}+۳\sqrt{۳}}{۵-\sqrt{۶}}\times\frac{۵+\sqrt{۶}}{۵+\sqrt{۶}}=\frac{۱۰\sqrt{۲}+۱۵\sqrt{۳}+(۲\sqrt{۲})(\sqrt{۶})+(۳\sqrt{۳})(\sqrt{۶})}{۵^۲-۶}$ (۳)

$\begin{cases}(۲\sqrt{۲})(\sqrt{۶})=۲\sqrt{۲}(\sqrt{۲}\times\sqrt{۳})=۴\sqrt{۳}\\(۳\sqrt{۳})(\sqrt{۶})=۳\sqrt{۳}(\sqrt{۳}\times\sqrt{۲})=۹\sqrt{۲}\end{cases}$

$A=\frac{۱۰\sqrt{۲}+۱۵\sqrt{۳}+۴\sqrt{۳}+۹\sqrt{۲}}{۱۹}=\frac{۱۹\sqrt{۲}+۱۹\sqrt{۳}}{۱۹}=\sqrt{۲}+\sqrt{۳}$

$\sqrt[۴]{۹}=\sqrt[۴]{۳^۲}\longrightarrow B=۲(\sqrt[۴]{۹}-۱)^{-۱}=\frac{۲}{\sqrt[۴]{۹}-۱}=\frac{۲}{\sqrt{۳}-۱}\times\frac{\sqrt{۳}+۱}{\sqrt{۳}+۱}=\frac{۲(\sqrt{۳}+۱)}{۳-۱}=\sqrt{۳}+۱$

$A-B=(\sqrt{۲}+\sqrt{۳})-(\sqrt{۳}+۱)=\boxed{\sqrt{۲}-۱}$

ج) $A=\frac{\sqrt{۲۷}-۱}{۴+\sqrt{۳}}=\frac{۳\sqrt{۳}-۱}{۴+\sqrt{۳}}\times\frac{۴-\sqrt{۳}}{۴-\sqrt{۳}}=\frac{۱۲\sqrt{۳}-۹-۴+\sqrt{۳}}{۴^۲-۳}=\frac{۱۳\sqrt{۳}-۱۳}{۱۳}=\sqrt{۳}-۱$

$B=(۲-\sqrt{۳})^{-۱}=\frac{۱}{۲-\sqrt{۳}}\times\frac{۲+\sqrt{۳}}{۲+\sqrt{۳}}=\frac{۲+\sqrt{۳}}{۲^۲-۳}=۲+\sqrt{۳}$

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$A+B=(\sqrt{۳}-۱)+(۲+\sqrt{۳})=\boxed{۱+۲\sqrt{۳}}$

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الف) $A = \sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1} \longrightarrow A^2 = (1+\sqrt{3}) + (\sqrt{3}-1) + 2\sqrt{(\sqrt{3}+1)(\sqrt{3}-1)}$ ④

$A^2 = 2\sqrt{3} + 2\sqrt{2} = 2(\sqrt{3}+\sqrt{2})$

$A = \sqrt{2(\sqrt{3}+\sqrt{2})}$

$$\frac{\sqrt{1+\sqrt{2}+\sqrt{\sqrt{3}-1}}}{\sqrt{\sqrt{3}-\sqrt{2}}} = \frac{\sqrt{2(\sqrt{3}+\sqrt{2})}}{\sqrt{\sqrt{3}-\sqrt{2}}} \times \frac{\sqrt{\sqrt{3}+\sqrt{2}}}{\sqrt{\sqrt{3}+\sqrt{2}}} = \frac{\sqrt{2(\sqrt{3}+\sqrt{2})^2}}{\sqrt{3}-2} = \sqrt{2}\times(\sqrt{3}+\sqrt{2}) = \sqrt{6}+2$$

$(\sqrt{6}+2)-2 = \sqrt{6}$

ب) $\sqrt[4]{2^{\frac{5}{2}}} \times \sqrt[4]{2\times 3^4} \times \sqrt{2^2\sqrt{2}} = 2^{\frac{5}{8}} \times 2^{\frac{1}{4}}\sqrt{2} \times \sqrt{\sqrt{2^{12}\times 2}} = \sqrt[3]{2^2} \times 3\sqrt{2} \times 2^{\frac{13}{12}}\sqrt{2}$

$= \sqrt[12]{2^8} \times 2\sqrt[12]{2^3} \times 2^{\frac{13}{12}}\sqrt{2} = \sqrt[12]{2^{12}} = 2$

$$\frac{2^n + 3^{n+1} + 2^{n+2} + 2^{n+3} + 2^{n+4} - 2^{n+5}}{2^{n-2} + 2^{n-1} + 2^n + 2^{n+1} + 2^{n+2} + 2^{n+3}} = 21$$ ⑤

$$\frac{2^n(1+2+3^2+3^3+2^4+3^5)}{2^{n-2}(1+2+2^2+2^3+2^4+2^5)} = 21 \longrightarrow \frac{2^n(1+2+9+27+81+243)}{2^{n-2}(1+2+4+8+16+32)} = 21$$

$$\frac{2^n \times 3^4 \times 5}{2^{n-2} \times 3 \times 5} = 21 \longrightarrow \frac{2^4}{2^{n-2}} = \frac{21\times 3}{5\times 3} \longrightarrow \frac{2^4}{2^{n-2}} = 4 \longrightarrow \frac{2^4}{2^n} = \frac{1}{4} \longrightarrow \frac{2^4}{2^n} = \frac{1}{4}$$

$\left(\frac{2}{3}\right)^n = \left(\frac{3}{2}\right)^2 \longrightarrow n = 2$

جواب ۶ در صفحه بعد ←

الف) $\left(a+\frac{1}{a}+\sqrt{2}\right)^2\left(a+\frac{1}{a}-\sqrt{2}\right)^2 = \left(\left(a+\frac{1}{a}\right)^2 - \left(\sqrt{2}\right)^2\right)^2$ ⑦

$= \left(a^2+\frac{1}{a^2}+2a\left(\frac{1}{a}\right)-2\right)^2 = \left(a^2+\frac{1}{a^2}\right)^2$ $\qquad 7-4\sqrt{3} = (2-\sqrt{3})^2$

$a = \sqrt[4]{7-4\sqrt{3}} = \sqrt[4]{(2-\sqrt{3})^2} = \sqrt{2-\sqrt{3}}$ $\qquad \left(a^2+\frac{1}{a^2}\right)^2 = \left(2-\sqrt{3}+\frac{1}{2-\sqrt{3}}\right)^2$

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$\frac{1}{2-\sqrt{3}} = 2+\sqrt{3}$ $\qquad (2-\sqrt{3}+2+\sqrt{3})^2 = 4^2 \longrightarrow 2^t = 4^2 = 2^4 \longrightarrow t = 4$

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$A = (a^2 + b^2 - 2ab)^2 (a^2 + b^2 + 2ab)^2$ ⑧

$= ((a-b)^2)^2 ((a+b)^2)^2 = ((a-b)^2 (a+b)^2)^2 = ((a^2-b^2)^2)^2 = (a^4+b^4-2a^2b^2)^2$ ☆

$$\begin{cases} a = \sqrt[4]{\sqrt{5}-2} \longrightarrow a^4 = \sqrt{5}-2 \;,\; a^2 = \sqrt{\sqrt{5}-2} \\ b = \sqrt[4]{\sqrt{5}+2} \longrightarrow b^4 = \sqrt{5}+2 \;,\; b^2 = \sqrt{\sqrt{5}+2} \end{cases}$$

$2a^2b^2 = 2\sqrt{\sqrt{5}-2} \times \sqrt{\sqrt{5}+2} = 2\sqrt{(\sqrt{5}-2)(\sqrt{5}+2)} = 2\sqrt{5-4} = 2\sqrt{1} = 2$

$A = ((\sqrt{5}-2)+(\sqrt{5}+2)-2\sqrt{2})^2 = (2\sqrt{5}-2\sqrt{2})^2 = 4(\sqrt{5}-\sqrt{2})^2$

$= 4(7+2\sqrt{10}) = 4(8-4\sqrt{3}) = 16(2-\sqrt{3})$

$\hookrightarrow t = 16$

$A = \sqrt[3]{\sqrt{4^3\sqrt{16}}} \left(\frac{1}{2}\right)^{-\frac{3}{2}} = \sqrt[3]{\sqrt{4^3 \times 16}} \; (2^{-1})^{-\frac{3}{2}} = \sqrt[3\times 2]{4^3 \times 4^2} \times 2^{\frac{3}{2}}$ ⑦

$= \sqrt[6]{4^5} \times 2^{\frac{3}{2}} = \sqrt{4} \times 2^{\frac{3}{2}} = 2^{\frac{1}{2}} \times 2^{\frac{3}{2}} = 2^{\frac{4}{2}} = 2^2$

$(2^9)^{-\frac{1}{2}} = (2 \times 2^3)^{-\frac{1}{2}} = (2^3)^{-\frac{1}{2}} = 2^{-1} = \frac{1}{2} = 0/5$

$\sqrt[7]{a} = 27 \times a^{\frac{15}{7}} \longrightarrow a^{\frac{1}{7}} = 27 \times a^{\frac{15}{7}} \longrightarrow \frac{a^{\frac{1}{7}}}{a^{\frac{15}{7}}} = 27$ ⑨

$a^{\frac{1}{7}-\frac{15}{7}} = 27 \longrightarrow a^{-\frac{14}{7}} = 27 \rightarrow a^{-2} = 27 \longrightarrow \sqrt{a^{-2}} = \sqrt{27} \longrightarrow a^{-1} = 3\sqrt{3} \longrightarrow \frac{1}{a} = 3\sqrt{3}$

$$\frac{\frac{1}{a}-3}{1+\sqrt{3}} = \frac{3\sqrt{3}-3}{\sqrt{3}+1} = \frac{3(\sqrt{3}-1)}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3(\sqrt{3}-1)^2}{3-1} = \frac{3(4-2\sqrt{3})}{2}$$

$$= \frac{3(4-2\sqrt{3})}{2} = 3(2-\sqrt{3}) = 6-3\sqrt{3}$$

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$(\sqrt{x+a}-\sqrt{x-b})(\sqrt{x+a}+\sqrt{x-b}) \rightarrow 2(\sqrt{x+a}+\sqrt{x-b})$

$\rightarrow (\sqrt{x+a})^2-(\sqrt{x-b})^2 \rightarrow 2(\sqrt{x+a}+\sqrt{x-b}) \rightarrow (x+a)-(x-b)$

$\rightarrow 2(\sqrt{x+a}+\sqrt{x-b}) \rightarrow a+b=2(\sqrt{x+a}+\sqrt{x-b}) \rightarrow \sqrt{x+a}+\sqrt{x-b}=\frac{a+b}{2}$

$\rightarrow \sqrt{x+a}+\sqrt{x-b}=\frac{a}{2}+2 \rightarrow \sqrt{x+a}+\sqrt{x-b}-2=\frac{a}{2}$]