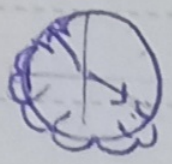
تمرين محلول ٢٠

① الف)



$6 \times 30 = 180$

$4 \times 6 = 24$

$\frac{54}{2} = 27 \Rightarrow 30 - 27 = 3$

$180 + 24 + 3 = 207$

$360 - 207 = 153$

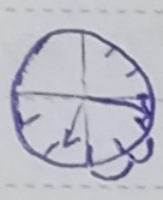
ب)

$|\omega_1 \omega_m - 30 h| = a$

$\omega_1 \omega \times \omega 4 - 30 \times 3 = 207$

$360 - 207 = 153$

② الف)



$2 \times 30 = 60$

$2 \times 6 = 12$

$\frac{18}{2} = 9$

$\Rightarrow 81$

ب)

$|\delta_1 \delta M - 30 h| = a$

$|\omega_1 \omega \times 18 - 30 \times 6| = 81$

③ الف)

$S = \frac{\alpha}{2} R^2 \Rightarrow \frac{\pi}{12} \times 9 = \frac{3}{4} \pi$

ب)

$\rho = \alpha R + 6 \Longrightarrow \frac{\pi}{6} \times 3 + 6 = \frac{\pi}{2} + 6$

۴) الف)

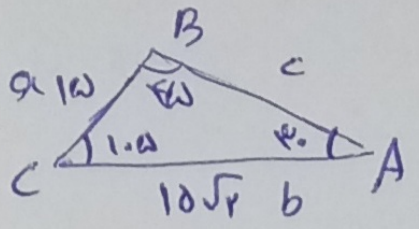
$$S = \frac{1}{2} ab \sin\alpha \Rightarrow \frac{1}{2} \times 10 \times 8 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$$

۱۰

$$C = \sqrt{a^2 + b^2 - 2ab\cos\alpha} = \sqrt{25 + 64 - 40} = \sqrt{49} = 7$$

$$P = 7 + 8 + 5 = 20$$

۳)



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{10}{\frac{1}{2}} = \frac{10\sqrt{2}}{\sin B} = \frac{c}{\sin C}$$

$$\sin B = \frac{\sqrt{2}}{2} \xrightarrow{\sin^{-1}} B = 45^\circ \rightsquigarrow \frac{45}{180} = \frac{\text{Rad}}{\pi}$$

$$180 - 45 = 105^\circ : C$$

$$\text{Rad} = \frac{45\pi}{180} = \frac{\pi}{4}$$

$$\hookrightarrow \frac{105}{180} = \frac{\text{Rad}}{\pi}$$

$$\text{Rad} = \frac{105\pi}{180} = \frac{7\pi}{12}$$

۴)

$$\frac{-\tan(\alpha) + 3\tan(\alpha)}{-\tan(\alpha) - \tan(\alpha)} = \frac{2\tan(\alpha)}{-2\tan(\alpha)} = -1$$

۵)

$$\frac{\frac{\pi}{12}}{\frac{\pi}{12}} = \frac{D}{180} \Rightarrow D = 15^\circ$$

$$\frac{2\tan\left(\frac{\pi}{2} - 15^\circ\right) + \tan\left(\frac{\pi}{2} + 15^\circ\right)}{2\tan(\pi - 15^\circ) - \tan\left(\frac{3\pi}{2} - 15^\circ\right)} =$$

$$\frac{2\cot(15^\circ) - \cot(15^\circ)}{-2\tan(15^\circ) - \cot(15^\circ)} = \frac{\frac{1}{a}}{-2a - \frac{1}{a}} = \frac{-1}{2a^2 + 1}$$

۸)

$$\frac{\sin^2\alpha+\cos^2\alpha+2\sin\alpha\cos\alpha}{\sin^2\alpha-\cos^2\alpha}+\frac{\sin^2\alpha+\cos^2\alpha-2\sin\alpha\cos\alpha}{\sin^2\alpha-\cos^2\alpha}=$$

$$\frac{2\sin^2\alpha+2\cos^2\alpha}{\sin^2\alpha-\cos^2\alpha}\xrightarrow[\text{تقسیم می‌کنیم}]{\text{همه را بر } \cos^2}$$

$$\frac{2\tan^2\alpha+2}{\tan^2\alpha-1}=3\longrightarrow 3\tan^2\alpha-3=2\tan^2\alpha+2$$

$$\tan^2\alpha=5$$

۹)

$$\xrightarrow[\text{تقسیم می‌کنیم}]{\text{همه را بر } \cos^2}\quad\frac{\tan^2\alpha-2+\frac{1}{\cos^2\alpha}}{\tan^2\alpha+2-\frac{1}{\cos^2\alpha}}=\frac{\tan^2\alpha-2+1+\tan^2\alpha}{\tan^2\alpha+2-1-\tan^2\alpha}=$$

$$\frac{2\tan^2\alpha-1}{1}=4\qquad \tan^2\alpha=\frac{5}{2}$$

۱۰) الف)

$$\cos 2\alpha=1-2\sin^2\alpha$$

$$\frac{\sqrt{2}}{2}=1-2\sin^2\alpha\Longrightarrow\sin^2\alpha=\frac{2-\sqrt{2}}{4}$$

$$\cos\alpha=\sqrt{\frac{1-2+\sqrt{2}}{4}}=\frac{\sqrt{\sqrt{2}-1}}{2}$$

ب)

$$-\frac{\sqrt{2}}{2}=1-2\sin^2\alpha$$

$$\sin\alpha=\frac{\sqrt{2+\sqrt{2}}}{2}$$