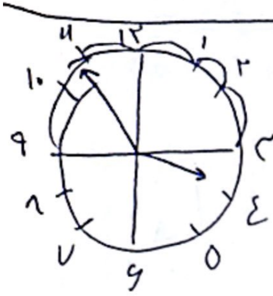
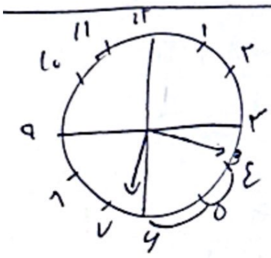
باران پاکزاد گروه ۱ دفترچه تکلیف ۱۹ ۲۰



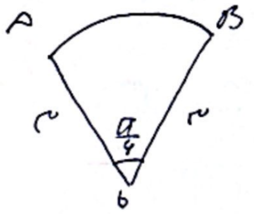
الف) $(4\times 30)+(54\div 2)+6 = 120+27+6 = \underline{153^\circ}$

ب) $\alpha = |(5{,}5\times 54)-(30\times 3)| = 207 \quad 360-207 = \underline{153^\circ}$ ۱



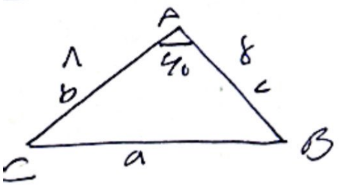
الف) $\underbrace{(2\times 30)}_{60}+\underbrace{(2\times 6)}_{12}+\underbrace{(18\div 2)}_{9} = \underline{81^\circ}$

ب) $\alpha = |(5{,}5\times 54)-(30\times 3)| = \underline{81^\circ}$ ۲



الف) $S_{OAB} = \frac{\alpha}{2}\cdot R^2 \Rightarrow \frac{\pi}{12}\times 9 = \boxed{\frac{3\pi}{4}}$ ۱

ب) $2R+\alpha R \Rightarrow \underbrace{(2\times 3)}_{6}+\underbrace{(\frac{\pi}{6}\times 3)}_{\frac{\pi}{2}} = \boxed{6+\frac{\pi}{2}}$ ۳

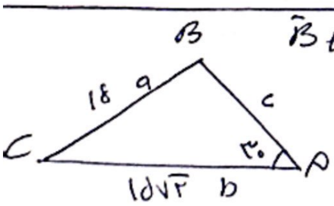


الف) $\Rightarrow S_{ABC} = \frac{1}{2} b.c.\sin 60 \Rightarrow \frac{1}{2}\times 8\times 5\times \frac{\sqrt{3}}{2} = 10\sqrt{3}$

ب) $\Rightarrow P_{ABC} \Rightarrow a^2 = b^2+c^2-2bc\cos 60 \Rightarrow$ ۴

$a^2 = 64+25-2(8\times 5\times \frac{1}{2}) \Rightarrow 64+25-40 = 49 \Rightarrow a = 7$

$P_{ABC} = 7+5+8 = \boxed{20}$



$\hat{B}+\hat{C} = 150 \Rightarrow \hat{A} = 30^\circ$

$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{15}{1/2} = 30 = \frac{15\sqrt{2}}{x} \Rightarrow 30x = 15\sqrt{2}$

$x = \frac{\sqrt{2}}{2} \Rightarrow \sin B = \frac{\sqrt{2}}{2} \quad B = \boxed{45} \Rightarrow \frac{\pi}{4} = \hat{B}$ ۵

$C+45 = \cancel{180}\ 150 \quad C = 105 \Rightarrow \frac{105}{180} = \frac{7}{12}\times \pi = \frac{7\pi}{12} = \hat{C}$

$$\frac{\tan(\pi-\alpha)+3\tan(\pi+\alpha)}{\tan(2\pi-\alpha)-\tan(2\pi+\alpha)} \Rightarrow \text{صورت} = \tan(\pi-\alpha)+3\tan(\pi+\alpha)$$

$$-\tan\alpha + 3\tan\alpha = +2\tan\alpha$$

$$\text{مخرج} = \tan(2\pi-\alpha) - \tan(2\pi+\alpha) \Rightarrow +\tan\alpha - (-\tan\alpha) = 2\tan\alpha \qquad \frac{2\tan\alpha}{2\tan\alpha} = \boxed{1}$$ ۶

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$$\frac{2\tan 75^\circ + \tan 105^\circ}{3\tan 165^\circ - \tan 255^\circ} = \frac{2\tan(\frac{\pi}{2}-15)+\tan(\frac{\pi}{2}+15)}{3\tan(\pi-15)-\tan(\frac{3\pi}{2}-15)} \Rightarrow$$

$$\frac{\pi}{12} \Rightarrow \frac{1}{12}\times 180 = 15^\circ \qquad \tan 15^\circ = a \qquad \cot 15 = \frac{1}{a}$$ ۷

$$\text{صورت} = 2\tan(\tfrac{\pi}{2}-15)+\tan(\tfrac{\pi}{2}+15)$$

$$+2\cot(15) + (-\cot(15)) \Rightarrow 2\cot 15^\circ - \cot 15 = \cot 15^\circ = \frac{1}{a}$$

$$\text{مخرج} = 3\tan(\pi-15) - \tan(\tfrac{3\pi}{2}-15) = -3\tan 15 - \cot 15 = -3a - \frac{1}{a} = \frac{-3a^2-1}{a}$$

$$\frac{\frac{1}{a}}{\frac{-3a^2-1}{a}} = \boxed{\frac{1}{-3a^2-1}}$$

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$$\frac{\sin x+\cos x}{\sin x-\cos x} \xrightarrow{\div\cos x} \frac{\tan x+1}{\tan x-1} + \frac{\tan x-1}{\tan x+1} = 3 \Rightarrow \frac{(\tan x+1)^2+(\tan x-1)^2}{\tan^2 x-1} = 3$$

$$\frac{\sin x-\cos x}{\sin x+\cos x} \xrightarrow{\div\cos} \qquad \frac{\tan^2 x+1+\cancel{2\tan x}+\tan^2 x+1-\cancel{2\tan x}}{\tan^2 x-1} = 3$$ ۸

$$\frac{2(\tan^2 x+1)}{\tan^2 x-1} = 3 \Rightarrow 3\tan^2 x - 3 = 2\tan^2 x + 2 \Rightarrow \boxed{\tan^2 x = 5}$$

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$$\frac{\sin^2 x - 2\cos^2 x + 1}{\sin^2 x + 2\cos^2 x - 1} = 4 \Rightarrow \frac{1-\cos^2 x - 2\cos^2 x + 1}{1-\cos^2 x + 2\cos^2 x - 1} \Rightarrow \frac{-3\cos^2 x + 2}{\cos^2 x} = 4$$

$$4\cos^2 x = -3\cos^2 x + 2 \qquad 7\cos^2 x = 2 \qquad \cos^2 x = \frac{2}{7} \qquad \sin^2 x = \frac{7}{7} - \frac{2}{7} = \frac{5}{7}$$ ۹

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{5}{\cancel{7}}}{\frac{2}{\cancel{7}}} = \frac{5}{2}$$

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$$\cos^2(22.5) = \frac{1+\cos 45}{2} \Rightarrow \frac{\frac{2}{2}+\frac{\sqrt{2}}{2}}{2} = \frac{\frac{2+\sqrt{2}}{2}}{\frac{2}{1}} = \frac{2+\sqrt{2}}{4} = \cos^2 22.5$$ ۱۰

$$\cos(22.5) = \frac{\sqrt{2+\sqrt{2}}}{2}$$

$$\sin^2(67.5) = \frac{1-\cos 135}{2} = \frac{\frac{2}{2}-(-\frac{\sqrt{2}}{2})}{2} \Rightarrow \sin(67.5) = \frac{\sqrt{2+\sqrt{2}}}{2}$$

چون ۶۷/۵ متمم ۲۲/۵ است پس سینوس آن برابر cos ۲۲/۵ است.