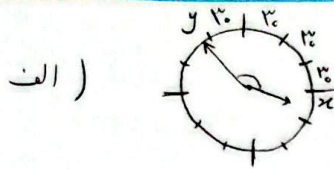


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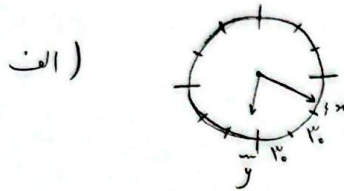
سوال ۱



$$x = \frac{d\alpha}{r} = 27 \quad y = 1 \times 4 = 4$$

$$\alpha = 30 + 30 + 30 + 30 + 27 + 4 = 151^\circ$$

$$\alpha = |d/dM - r.H| \rightarrow |d/d(27) - r.(4)| = |27 - 9| = 18 \rightarrow 27 - 9 = 18$$



$$y = \frac{12}{r} = 9^\circ \quad x = 2 \times 4 = 12$$

$$\alpha = 12 + 30 + 30 + 9 = 81^\circ$$

$$\alpha = |d/dM - r.H| = |d/d(12) - r.(4)| = |9 - 12| = 3$$

$$S = \frac{\alpha}{r} R^2 \rightarrow \frac{\pi}{4} \times r^2 = \frac{\pi}{12} \times 9 = \frac{3\pi}{4}$$

$$\alpha R = \alpha R + rR \rightarrow \frac{\pi}{4} \times r + r \times r = \frac{\pi}{4} + 4$$



$$S = \frac{1}{2} AB \times AC \times \sin 40^\circ = \frac{1}{2} \times d \times 1 \times \frac{\sqrt{3}}{2} = \frac{1.5\sqrt{3}}{2}$$

$$a = \sqrt{b^2 + c^2 - 2bc \cos A} = \sqrt{1^2 + d^2 - 2(1)(d)(\frac{1}{2})} = \sqrt{1 + d^2 - d} = \sqrt{d^2 - d + 1} = \sqrt{4 - 2 + 1} = \sqrt{3} = \sqrt{3}$$

$$\hat{A} = 120 - 120 = 0^\circ$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} \rightarrow \frac{1}{\frac{1}{2}} = \frac{1.5\sqrt{3}}{\sin B} \rightarrow \sin B = \frac{1.5\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$$\sin B = \frac{\sqrt{3}}{2} \rightarrow \hat{B} = 60^\circ \rightarrow \text{چون زاویه } 60^\circ \text{ و } 120^\circ \text{ با هم } 180^\circ \text{ می شود}$$

$$\hat{C} = 120 - 60 = 60$$

$$\hat{B} : \frac{60}{120} = \frac{\text{rad}}{\pi} \rightarrow \text{rad} = \frac{60\pi}{120} = \frac{\pi}{2}$$

$$\hat{C} : \frac{60}{120} = \frac{\text{rad}}{\pi} \rightarrow \text{rad} = \frac{60\pi}{120} = \frac{\pi}{2}$$

$$\frac{\tan(\pi - \alpha) + 3 \tan(\pi + \alpha)}{\tan(2\pi - \alpha) - \tan(2\pi + \alpha)} = \frac{-\tan \alpha + 3 \tan \alpha}{-\tan \alpha - \tan \alpha} = \frac{2 \tan \alpha}{-2 \tan \alpha} = -1$$

$$\frac{\text{rad}}{\pi} = \frac{D}{120} \rightarrow \frac{\frac{\pi}{12}}{\pi} = \frac{D}{120} \rightarrow D = \frac{120}{12} = 10$$

$$A = \frac{2 \tan(\frac{\pi}{2} - \frac{\pi}{12}) + \tan(\frac{\pi}{2} + \frac{\pi}{12})}{2 \tan(\pi - \frac{\pi}{12}) - \tan(2\pi - \frac{\pi}{12})} = \frac{2 \cot \frac{\pi}{12} - \cot \frac{\pi}{12}}{-2 \tan \frac{\pi}{12} - \cot \frac{\pi}{12}} = \frac{\frac{1}{a}}{-2a - \frac{1}{a}} = \frac{\frac{1}{a}}{-\frac{2a^2 + 1}{a}} = \frac{1}{-2a^2 - 1}$$

$$\frac{1}{\sin^2 \alpha} \rightarrow \frac{(\sin \alpha + \cos \alpha)^2 + (\sin \alpha - \cos \alpha)^2}{(\sin \alpha - \cos \alpha)(\sin \alpha + \cos \alpha)} = \frac{\sin^2 \alpha + \cos^2 \alpha + 2\sin \alpha \cos \alpha + \sin^2 \alpha + \cos^2 \alpha - 2\sin \alpha \cos \alpha}{\sin^2 \alpha - \cos^2 \alpha} = \quad (1 \text{ سوال})$$

$$\frac{1+1}{\sin^2 \alpha - \cos^2 \alpha} = \frac{2}{\sin^2 \alpha - \cos^2 \alpha} \xrightarrow{\div \cos^2 \alpha} \frac{\frac{2}{\cos^2 \alpha}}{\tan^2 \alpha - 1} = \frac{2(1+\tan^2 \alpha)}{\tan^2 \alpha - 1} = 2 \rightarrow 2 + 2\tan^2 \alpha = \tan^2 \alpha - 2$$

$$\downarrow$$

$$\boxed{\tan^2 \alpha = 4}$$

$$\frac{\tan^2 \alpha - 2 + \frac{1}{\cos^2 \alpha}}{\tan^2 \alpha + 2 - \frac{1}{\cos^2 \alpha}} = \frac{\tan^2 \alpha - 2 + \tan^2 \alpha + 1}{\tan^2 \alpha + 2 - \tan^2 \alpha - 1} = 2\tan^2 \alpha - 1 \quad (2 \text{ سوال})$$

$$2\tan^2 \alpha - 1 = 2 \rightarrow 2\tan^2 \alpha = 3 \rightarrow \tan^2 \alpha = \frac{3}{2} = \frac{r}{d}$$

$$\text{الف) } \cos^2 \alpha = \frac{1 + \cos^2 \alpha}{2} \rightarrow \cos^2 (r/d) = \frac{1 + \cos^2 d}{2} = \frac{1 + \frac{\sqrt{r}}{r}}{2} = \frac{\frac{r + \sqrt{r}}{r}}{2} = \frac{r + \sqrt{r}}{2r} \quad (1 \text{ سوال})$$

$$\rightarrow \cos (r/d) = \sqrt{\frac{r + \sqrt{r}}{2r}} = \frac{\sqrt{r + \sqrt{r}}}{\sqrt{2r}}$$

$$\text{ب) } \sin^2 \alpha = \frac{1 - \cos^2 \alpha}{2} \quad \sin^2 (r/d) = \frac{1 - \cos^2 d}{2} = \frac{\frac{r - \sqrt{r}}{r}}{2} = \frac{r - \sqrt{r}}{2r} \rightarrow \sin (r/d) = \frac{\sqrt{r - \sqrt{r}}}{\sqrt{2r}}$$

لذا: $\cos (r/d) = \sin (r/d)$ و $\sin (r/d) = \cos (r/d)$ و $\sin (r/d) = \cos (r/d)$ و $\sin (r/d) = \cos (r/d)$