①

الف) $\tan(3\pi-\frac{\pi}{4})+(\sin(4\pi-\frac{\pi}{4})\cos(3\pi+\frac{\pi}{4}))=$

$-\tan\frac{\pi}{4}\times\cos\sin\frac{\pi}{4}=-\sin^2\frac{\pi}{4}=\boxed{-\frac{1}{2}}$

ب) $\tan(3\pi-\frac{\pi}{6})\sin(4\pi-\frac{\pi}{3})+\cos(3\pi+\frac{\pi}{3})$

$(-\tan\frac{\pi}{6})(-\sin\frac{\pi}{3})+(-\cos\frac{\pi}{3})=(-\frac{\sqrt3}{3}\times-\frac{\sqrt3}{2})+(-\frac{1}{2})=\boxed{0}$

②

همه رو بر sin تقسیم می‌کنیم $\rightarrow \frac{3\cot x-1}{\cot x+1}=\frac{12-1}{4+1}=\boxed{\frac{11}{5}}$

③

$\sin\alpha=2\cos\alpha\Rightarrow\tan\alpha=2\qquad 1+\tan^2\alpha=\frac{1}{\cos^2\alpha}\rightarrow 1+4=\frac{1}{\cos^2\alpha}\rightarrow\cos\alpha=\boxed{-\frac{1}{\sqrt5}}$

④

الف) ربع ۲ و $\sin\alpha=\frac{\sqrt2}{10}\Rightarrow\cos(3\frac{\pi}{4}+\alpha)$

$\sin^2\alpha\rightarrow\cos^2\alpha-1\rightarrow(\frac{\sqrt2}{10})^2+\cos^2\alpha=1\rightarrow\cos\alpha=-\frac{7\sqrt2}{10}$

$\cos(3\frac{\pi}{4}+\alpha)=\cos3\frac{\pi}{4}\cos\alpha-\sin3\frac{\pi}{4}\sin\alpha=\frac{7}{10}-\frac{1}{10}=\boxed{\frac{6}{10}}$

($\cos3\frac{\pi}{4}\rightarrow-\frac{\sqrt2}{2}$, $\cos\alpha\rightarrow-\frac{7\sqrt2}{10}$, $\sin3\frac{\pi}{4}\rightarrow\frac{\sqrt2}{2}$, $\sin\alpha\rightarrow\frac{\sqrt2}{10}$)

ب) ربع ۳ و $\tan\alpha=\frac{1}{7}\rightarrow\sin(\frac{13\pi}{4}+\alpha)\qquad 1+\frac{1}{49}=\frac{1}{\cos^2}\rightarrow\cos\alpha=\frac{7}{5\sqrt2}$ و $\sin\frac{5\pi}{4}+\alpha)$

$\sin(5\frac{\pi}{4}+\alpha)=\sin5\frac{\pi}{4}\cos\alpha+\cos5\frac{\pi}{4}\sin\alpha\rightarrow-\frac{\sqrt2}{2}\cos\alpha-\frac{\sqrt2}{2}\sin\alpha=-\frac{\sqrt2}{2}(\cos+\sin)$

$\boxed{\cos=\frac{7\sqrt2}{10}}$ و $\sin=\tan\times\cos=\boxed{\frac{\sqrt2}{10}}\qquad -\frac{\sqrt2}{2}(\frac{7\sqrt2}{10}+\frac{\sqrt2}{10})=\boxed{-\frac{4}{5}}$

⑤

$\sin^2x+\cos^2x+\sin^2x=\frac{4}{3}\Rightarrow\sin^2x=\frac{1}{3}\rightarrow1+\cot^2x=\frac{1}{\sin^2x}\Rightarrow\cot^2x=2$

$\tan^2x=\frac{1}{\cot^2x}\rightarrow\tan^2x=\boxed{\frac{1}{2}}$

⑥

الف) $\sqrt3, 6, \alpha\qquad S=4/5$

$4/5=\frac{1}{2}\times ab\times\sin\alpha\rightarrow\frac{1}{2}\times\sqrt3\times6\times\sin\alpha=4/5\rightarrow\sin\alpha=\frac{\sqrt3}{2}\rightarrow 0<\alpha<180$ یا $0<\alpha<\pi$

$\frac{\pi}{3}\rightarrow60\qquad\frac{2\pi}{3}\rightarrow120\qquad\frac{120}{60}=2=\frac{\frac{2\pi}{3}}{\frac{\pi}{3}}=\boxed{2}$

x
$180 - 30$

(۷)

$ab\sin\alpha = 54 \rightarrow ab(\sin 150) = 54 \rightarrow ab(\frac{1}{2}) = 54 \Rightarrow ab = 108$

$a = \frac{2}{3}b \Rightarrow (\frac{2}{3}b)(b) = 108 \rightarrow \frac{2}{3}b^2 = 108 \rightarrow b^2 = 162$ و $b = 9\sqrt{2}$ و $a = 6\sqrt{2}$

محیط $= 2(9\sqrt{2} + 6\sqrt{2}) = 30\sqrt{2}$

(۸)

$S(ABC) - S(ADE) = 1/75$

$S = \frac{1}{2}ab\sin\Theta$ $\qquad \frac{1}{2}(AB)(AC)\sin A = \frac{1}{2} \times 5 \times 7 \times \sin A = 17/5 \sin A$

$\frac{1}{2}(AD)(AE)\sin A = \frac{1}{2} \times 7 \times 4 \times \sin A = 14\sin A \qquad 17/5\sin A - 14\sin A = 3/5\sin A = 1/75$

$1 + \cot^2 A = \frac{1}{\sin^2\alpha} \Rightarrow \cot^2 A = 3 \rightarrow \tan A = \frac{1}{\sqrt{3}} \qquad \sin A = \frac{1}{2}$

(۹)

$2BN = 3NC \rightarrow \frac{BN}{NC} = \frac{3}{2}$

$BC = BN + NC = 3k + 2k = 5k \rightarrow \frac{BM}{BC} = \frac{3k}{5k} = \frac{3}{5}$

$\frac{S\,BMN}{S\,ABC} = \frac{BM \times BN}{BA \times BC} = \frac{1}{3} \rightarrow \frac{1}{3} = \frac{BM}{BA} \times \frac{3}{5} \rightarrow \frac{BM}{BA} = \frac{5}{9}$

$\frac{BM}{BM + AM} = \frac{5}{9} \rightarrow \frac{BM}{AM} = \frac{5}{9-5} = \frac{5}{4}$

(۱۰)

☆ $\frac{1}{\sqrt{\cos^2\alpha}} - \tan\alpha = \frac{1 + \sin\alpha}{|\cos\alpha|} \Rightarrow \cos\alpha \neq 0$ پس $\rightarrow \cot\alpha \neq 0$ و $\sin\alpha < 0$

* $\frac{|\sin\alpha|}{\cos\alpha} = -\frac{1}{\cot\alpha} \rightarrow |\sin\alpha| = -\sin\alpha \Rightarrow \sin < 0$ ربع ۳ یا ۴

$\cot\alpha \rightarrow \frac{\cos}{\sin}$

هردو منفی پس ربع سوم

* $\frac{-\sin\alpha}{\cos\alpha} = \frac{-\sin\alpha}{\cos\alpha} \rightarrow \sin\alpha < 0$ و $\cos\alpha \neq 0$

☆ $\frac{1 - (1 + \sin\alpha)}{|\cos\alpha|} = \tan\alpha \rightarrow \frac{1 - 1 - \sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha} \rightarrow \frac{-\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha} \rightarrow \cos\alpha < 0$