تمرین سری سوم ۲۰

۱) الف) $\tan\left(3\pi - \frac{\pi}{4}\right) + \sin\left(4\pi - \frac{\pi}{4}\right) \cos\left(3\pi + \frac{\pi}{4}\right)$

$= -\tan\left(\frac{\pi}{4}\right) + -\sin\left(\frac{\pi}{4}\right) \cdot -\cos\left(\frac{\pi}{4}\right) =$

$-1 + \frac{-\sqrt{2}}{2} \times -\frac{\sqrt{2}}{2} = -\frac{1}{2}$

روز بزرگداشت مولوی

ش ی د س چ پ ج ش ج پ چ س د ی ش د س چ پ ج ش ج پ چ س د ی ش

۱ ۲ ۳ ۴ ۵ ۶ ۷ ۸ ۹ ۱۰ ۱۱ ۱۲ ۱۳ ۱۴ ۱۵ ۱۶ ۱۷ ۱۸ ۱۹ ۲۰ ۲۱ ۲۲ ۲۳ ۲۴ ۲۵ ۲۶ ۲۷ ۲۸ ۲۹ ۳۰

ب) $\tan\left(3\pi - \frac{\pi}{4}\right)\sin\left(4\pi - \frac{\pi}{3}\right) + \cos\left(3\pi + \frac{\pi}{3}\right)$

$-\tan\left(\frac{\pi}{4}\right)\cdot -\sin\left(\frac{\pi}{3}\right) + -\cos\left(\frac{\pi}{3}\right)$

$-\frac{1}{\sqrt{3}} \times -\frac{\sqrt{3}}{2} + -\frac{1}{2} = \boxed{0}$

(۲)

$$\frac{\frac{3\cos\alpha}{\sin\alpha} - \frac{\sin\alpha}{\sin\alpha}}{\frac{\cos\alpha}{\sin\alpha} + \frac{\sin\alpha}{\sin\alpha}} = \frac{12-1}{4+1} = \frac{11}{5}$$

$\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$

(۳)

$\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \frac{2\cos\theta}{\cos\theta} = 2$

$1 + \tan^2\theta = \frac{1}{\cos^2\theta}$

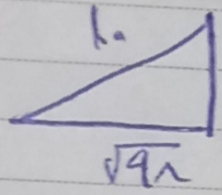
$1 + 4 = \frac{1}{\cos^2\theta} \Rightarrow \cos^2\theta = \frac{1}{5}$

$\cos\theta = -\frac{\sqrt{5}}{5}$

گزینه صفر است زیرا در ربع سوم مثبت است.

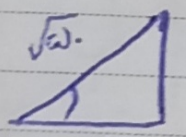
(٤)

ب) $\cos\left(\frac{\pi}{4}+\alpha\right) = \cos\frac{\pi}{4}\times\cos\alpha - \sin\frac{\pi}{4}\sin\alpha$



$\cos = \frac{-\sqrt{٩٨}}{١٠}$

$\frac{-\sqrt{٢}}{٢}\left(\frac{-\sqrt{٩٨}+\sqrt{٢}}{١٠}\right) = \frac{-\sqrt{٢}}{٢}\left(\frac{-٨\sqrt{٢}}{١٠}\right) = \frac{٤}{٥}$

ج) 

$\sin\alpha = \frac{\sqrt{٥٠}}{٥٠}$

$\cos\alpha = \frac{٧\sqrt{٥٠}}{٥٠}$

$\sin\left(\frac{٥\pi}{٤}+\alpha\right) = \sin\frac{٥\pi}{٤}\cos\alpha + \cos\frac{٥\pi}{٤}\sin\alpha$

$\frac{-\sqrt{٢}}{٢}\times\frac{٧\sqrt{٥٠}}{٥٠} + \frac{-\sqrt{٢}}{٢}\times\frac{\sqrt{٥٠}}{٥٠} = \frac{-٤}{٥}$

$\sin^2\theta + \cos^2\theta + \sin^2\theta = \frac{3}{2} \Rightarrow \sin^2\theta = \frac{1}{2}$ (۵)

$\sin^2\theta + \cos^2\theta = 1$

$\frac{1}{2} + \cos^2\theta = 1$

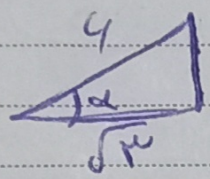
$\cos^2\theta = \frac{1}{2}$

$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha}$

$1 + \tan^2\theta = \frac{2}{1}$

$\tan^2\theta = 1$

(۶)



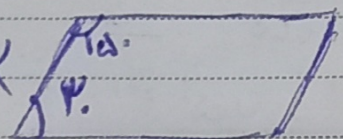
$S = \frac{1}{2} ab \sin\alpha$

$S = \frac{1}{2} \times 6 \times \sqrt{3} \times \sin\alpha = 4.5$

$\sin\alpha = \frac{\sqrt{3}}{2}$ ⇝ 120° ، 60° نظیر

$\frac{\alpha_{max}}{\alpha_{min}} = \frac{120}{60} = 2$

(۷)



$S = ab \sin\alpha$

$2K \times 3K \times \frac{1}{2} = 54$

$K = \sqrt{18} = 3\sqrt{2}$

$P = 2(6\sqrt{2} + 9\sqrt{2}) = 30\sqrt{2}$

(۸)

$S_{ABC} = \frac{1}{2} \times AB \times AC \times \sin A =$

$\frac{1}{2} \times 5 \times 7 \times \sin A = \frac{35}{2} \sin A$

$S_{ADE} = \frac{1}{2} \times AD \times AE \times \sin A =$

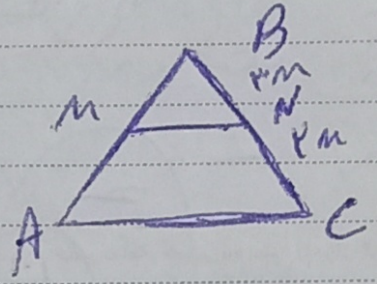
$\frac{1}{2} \times 7 \times 4 \times \sin A = \frac{28}{2} \sin A$

$S_1 - S_2 \Rightarrow \frac{7}{2} \sin A = \frac{175}{100}$

$\sin A = \frac{1}{2}$

$\xrightarrow{\text{حاد } A \text{ و } \sin A = \frac{1}{2}} A = \frac{\pi}{4} \rightarrow \tan A = \frac{\sqrt{3}}{3}$

(۹)



$S_{ABC} = \frac{1}{2} \times AB \times 5 \times \sin B$

$S_{BMN} = \frac{1}{2} \times BM \times 3m \times \sin B$

$\frac{5}{3} \times \frac{AB}{BM} = 3 \quad \frac{AB}{BM} = \frac{9}{5} \rightarrow \frac{BM}{AM} = \frac{5}{4}$

(۱۰)

$\frac{1}{\sqrt{\cos^2 \alpha}} - \tan \alpha = \frac{1 + \sin \alpha}{|\cos \alpha|}$

$\frac{1}{|\cos \alpha|} - \frac{1 + \sin \alpha}{|\cos \alpha|} = \tan \alpha$

$$\frac{-\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha} \Rightarrow \cos\alpha < 0$$

$$\frac{|\sin\alpha|}{\cos\alpha} = -\tan\alpha$$

$\hookrightarrow \sin\alpha > 0$

ناحیه سوم