بسط قرامانی و محاسبه $\cos, \sin, \tan$

①

الف) $\tan\frac{11\pi}{4} + \sin\frac{15\pi}{4}\cos\frac{13\pi}{4}$

$\tan(2\pi+\frac{3\pi}{4}) + \sin(2\pi+\frac{3\pi}{4})\cos(2\pi+\frac{\pi}{4})$

$\tan(\frac{3\pi}{4}) + \sin(\frac{3\pi}{4})\cos(\frac{\pi}{4})$

$-1 + \frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2} = -1+\frac{1}{2} = \boxed{-\frac{1}{2}}$

ب) $\tan\frac{17\pi}{4}\sin\frac{11\pi}{3} + \cos\frac{10\pi}{3}$

$\tan(2\pi+\frac{9\pi}{4})\sin(2\pi+\frac{2\pi}{3}) + \cos(2\pi+\frac{\pi}{3})$

$\tan(\frac{9\pi}{4})\sin(\frac{2\pi}{3}) + \cos(\frac{\pi}{3})$

$\boxed{0} = \frac{1}{2} + \underbrace{-\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}}_{-\frac{1}{2}}$

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②

$\frac{2\cos-\sin}{\cos+\sin} \xrightarrow{\div\cos} \frac{2(1)-1}{1+1} = \boxed{\frac{1}{2}}$

$\cot = \frac{\cos}{\sin} \to \begin{cases}\sin\to 1\\ \cos\to 1\end{cases}$

(٣)

$\sin = 2\cos \rightarrow \cos, \sin$ ٢ⓞ

$\sin^2 + \cos^2 = 1 \rightarrow 4\cos^2 + \cos^2 = 5\cos^2 = 1$

$\cos^2 = \frac{1}{5}$

$\cos = \pm\frac{1}{\sqrt{5}} \rightarrow \boxed{-\frac{\sqrt{5}}{5}}$ ٢ⓞⓒ

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(٤)

ب) $\cos\left(\frac{11\pi}{4} \ominus \alpha\right) \rightarrow \cos\frac{11\pi}{4}\cos\alpha \oplus \sin\frac{11\pi}{4}\sin\alpha$

$\cos(2\pi + \frac{3\pi}{4})$ $\sin(2\pi + \frac{3\pi}{4})$

$\sin\alpha = \frac{\sqrt{2}}{10}$

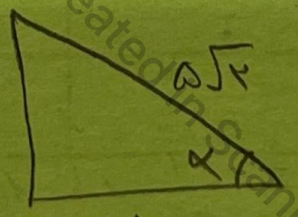
$\cos^2\alpha = 1 - \sin^2\alpha = 1 - \frac{2}{100} = \frac{98}{100} \rightarrow \cos\alpha = \pm\frac{\sqrt{98}}{10} = \pm\frac{7\sqrt{2}}{10}$ ٢ⓞ

$\Rightarrow -\frac{\sqrt{2}}{2} \times -\frac{7\sqrt{2}}{10} + \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{10} \Rightarrow \boxed{\frac{3}{5}}$

ج) $\sin\left(\frac{13\pi}{4} + \alpha\right) \rightarrow \sin\frac{12\pi}{4}\cos\alpha + \sin\alpha\cos\frac{\pi}{4}$

$\sin(2\pi + \frac{\pi}{4})$ $\sin(2\pi + \frac{\pi}{4})$

$\tan = \frac{1}{7}$



$\sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$

$\sin\alpha = \frac{1}{5\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{10}$

$\cos\alpha = \frac{7}{5\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{7\sqrt{2}}{10}$

$\Rightarrow \frac{\sqrt{2}}{2} \times \frac{7\sqrt{2}}{10} + \frac{\sqrt{2}}{10} \times \frac{\sqrt{2}}{2} = \frac{4}{5}$

③

$2\sin^2 + \cos^2 = \frac{5}{2}$

$\sin^2 + \underbrace{\sin^2 + \cos^2}_{1} = \frac{5}{2} \rightarrow \sin^2 = \frac{5}{2} - 1 = \frac{3}{2}$

$\rightarrow \frac{3}{2} + \cos^2 = \frac{5}{2} \rightarrow \cos^2 = \frac{2}{2}$ $\quad \tan^2 = \frac{\frac{3}{2}}{\frac{2}{2}} = \boxed{\frac{1}{2}}$

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⑥

$S = \frac{1}{2} \times 6 \times \sqrt{4} \times \sin\alpha = 6\sqrt{3}$

$\sin\alpha = \frac{\sqrt{3}}{2}$ (اندازه‌ی زاویه)

$\alpha = 60$ $\quad \frac{\pi}{3}$

$\alpha = 120$ $\quad \frac{2\pi}{3}$ (بیشترین)

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⑦

$S = 2 \times 4 \times \sin\alpha = 6\sqrt{2}$

$\sin\alpha = \frac{\sqrt{2}}{2}$

$\alpha = 45$ $\quad \frac{\pi}{4}$

$\alpha = 135$ $\quad \frac{3\pi}{4}$ (بیشترین)

$3k^2 = 54$

$k^2 = 18$

$k = 3\sqrt{2}$

$P = 2(9\sqrt{2} + 6\sqrt{2}) = 30\sqrt{2}$

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⑪

$S_{ADE} = \frac{1}{2} \times 7 \times 4 \times \sin A = 14k$

$S_{ABC} = \frac{1}{2} \times 7 \times 5 \times \sin A = 5k$

$5k - 4k = k = 1.75$

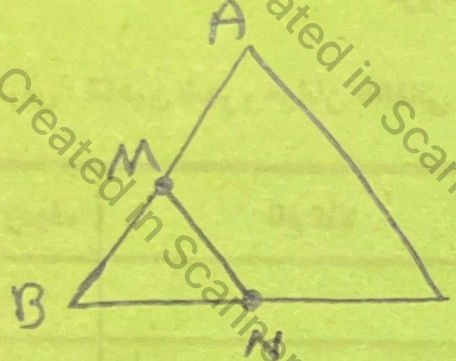
$\frac{1}{2} \times 7 \times 4 \times \sin A = 7$

$2\sin A = 1$

$\sin A = \frac{1}{2}$

$\cos A = \frac{\sqrt{3}}{2}$ $\quad \tan A = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$

(۹)



$۲BN = ۲NC \rightarrow BN = \frac{۲}{۲}NC$

$S_{ABC} = ۲^{۲} S_{BMN}$

$\frac{۱}{۲} \times AB \times BC \times \sin B = \frac{۲}{۲} \times BM \times BN \times \sin B$

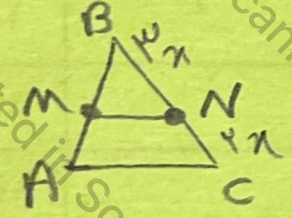
$BC = BN + NC \rightarrow BC = BN + \frac{۲}{۲}BN = \frac{۳}{۲}BN$

$AB = AM + BM \rightarrow \frac{۹}{۵}BN = BM = AM \rightarrow AM = \frac{۴}{۳}BM$

$\frac{BM}{AM} = \frac{۳}{۴}$

$AB \times ۵M \times \frac{۱}{۲} \times \sin B = ۲(\frac{۲}{۲} \times \sin \times ۲n \times BM)$

$۵BM + ۵AM = ۹BM \Rightarrow ۵AM \rightarrow \frac{BM}{AM} = \frac{۵}{۴}$



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(۱۰)

$\frac{۱}{\sqrt{\cos^{۲}}} - \tan = \frac{۱+\sin}{|\cos|} \rightarrow \frac{۱}{|\cos|} - \frac{۱+\sin}{|\cos|} = \frac{\sin}{\cos}$

$\frac{|\cos|}{\cos}$

$\frac{۱+۲-\sin}{|\cos|} = \frac{\sin}{\cos}$

$\frac{|\sin|}{\cos\alpha} = -\frac{\sin}{\cos} \rightarrow |\sin| = -\sin \rightarrow \sin < ۰$

$-|\cos| = \cos$

$\cos < ۰$

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