سونیا دهقانی

۱- الف) $\tan\frac{11\pi}{4}+\sin\frac{15\pi}{4}\cos\frac{13\pi}{4}$

$\tan\left(2\pi+\frac{3\pi}{4}\right)+\sin\left(4\pi-\frac{\pi}{4}\right)\cos\left(2\pi+\frac{5\pi}{4}\right)=-1+\left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)=-1+\frac{1}{2}$

$=-\frac{1}{2}$

ب) $\tan\frac{17\pi}{4}\sin\frac{11\pi}{3}+\cos\frac{10\pi}{3}$

$\tan\left(3\pi-\frac{\pi}{4}\right)\sin\left(3\pi+\frac{2\pi}{3}\right)+\cos\left(3\pi+\frac{\pi}{3}\right)=-\frac{\sqrt{3}}{3}\times-\frac{\sqrt{3}}{2}+-\frac{1}{2}=\frac{3}{4}-\frac{3}{4}=0$

۲- $\cot x=4 \to \dfrac{\frac{3\cos x}{\sin x}-\frac{\sin x}{\sin x}}{\frac{\cos x}{\sin x}+\frac{\sin x}{\sin x}}=\frac{12-1}{4+1}=\frac{11}{5}$

۳- $\sin\alpha=2\cos\alpha$, ربع سوم $\to \cos\alpha=?$

$\sin^2\alpha+\cos^2\alpha=1 \to (2\cos\alpha)^2+\cos^2\alpha=1 \to 4\cos^2\alpha+\cos^2\alpha=1$

ربع سوم

$\to \cos^2\alpha=\frac{1}{5} \to \cos\alpha=-\frac{\sqrt{5}}{5} \quad \to 5\cos\alpha^2=1$

۴- الف) ربع ۲ , $\sin\alpha=\frac{\sqrt{2}}{10} \to \cos\left(\frac{11\pi}{4}+\alpha\right)=?$

$\cos\left(2\pi+\frac{3\pi}{4}\right)\alpha \Rightarrow \left(\frac{3\pi}{4}+\alpha\right)$

$\sin^2\alpha+\cos^2\alpha=1 \to \left(\frac{\sqrt{2}}{10}\right)^{\cancel{2}}+\cos^2\alpha=1$

$\frac{1}{50}$

$\cos^2\alpha=\frac{50}{50}-\frac{1}{50}=\frac{49}{50} \to \cos\alpha=-\frac{7\sqrt{2}}{10}$

$\cos\left(\frac{3\pi}{4}+\alpha\right)=\cos\frac{3\pi}{4}\cos\alpha-\sin\frac{3\pi}{4}\sin\alpha=\frac{7}{10}-\frac{1}{10}=\frac{6}{10}=\frac{3}{5}$

ب) $1+\tan^2\alpha=\frac{1}{\cos^2\alpha} \to 1+\left(\frac{1}{7}\right)=\frac{1}{\cos^2} \to \frac{1}{\cos\alpha}=\frac{5\sqrt{2}}{7} \quad \sin\left(\frac{5\pi}{4}+\alpha\right)$

$\cos\alpha=\frac{7}{5\sqrt{2}}=\frac{7\sqrt{2}}{10} \to -\frac{\sqrt{2}}{2}(\cos\alpha+\sin\alpha) \to \cos\alpha=\frac{7\sqrt{2}}{10}$

$\sin\alpha=\tan\alpha\times\cos\alpha=\frac{1}{7}\times\frac{7\sqrt{2}}{10}=\frac{\sqrt{2}}{10} \to -\frac{\sqrt{2}}{2}\left(\frac{7\sqrt{2}}{10}+\frac{\sqrt{2}}{10}\right)=-\frac{\sqrt{2}}{2}\times\frac{4\sqrt{2}}{5}=-\frac{4}{5}$

۵-

$۲\sin^۲x + \cos^۲x = \frac{۴}{۳} \rightarrow \sin^۲x + \sin^۲x + \cos^۲x = \frac{۴}{۳} \rightarrow \sin^۲x + ۱ = \frac{۴}{۳}$

$\sin^۲x = \frac{۱}{۳}$ $\quad \frac{۱}{۳} + \cos^۲x = ۱ \Rightarrow \cos^۲x = \frac{۳}{۳} - \frac{۱}{۳} = \frac{۲}{۳}$ $\quad \tan^۲ = \frac{\sin^۲x}{\cos^۲x} = \frac{\frac{۱}{۳}}{\frac{۲}{۳}} = \frac{۱}{۲}$

۶-

$\sqrt{۳},\ ۶,\ \alpha \qquad S = ۴,۵$

$۴,۵ = \frac{۱}{۲} ab\sin\alpha \Rightarrow \frac{۱}{۲} \times \sqrt{۳} \times ۶ \times \sin\alpha = ۴,۵ \Rightarrow ۳\sqrt{۳}\sin\alpha$

$\sin\alpha = \frac{۴,۵}{۳\sqrt{۳}} = \frac{۱۳,۵\sqrt{۳}}{۲۷} = \frac{\sqrt{۳}}{۲} \rightarrow ۰ < \alpha < ۹۰$ یا $۰ < \alpha < \pi$

$\underbrace{\frac{\pi}{۳} \rightarrow ۶۰}_{min} \qquad \underbrace{\frac{۲\pi}{۳} \rightarrow ۱۲۰}_{max} \qquad \frac{۱۲۰}{۶۰} = ۲$ یا $\quad \frac{\frac{۲\pi}{۳}}{\frac{\pi}{۳}} = ۲$

۷-

$S = ۵۴ \qquad \frac{a}{b} = \frac{۲}{۳}$

$۵۴ = ab\sin\alpha \rightarrow ۵۴ = ab(\sin ۱۵۰)$

$\rightarrow \sin(۱۸۰ - ۳۰) = \sin ۳۰ = \frac{۱}{۲}$

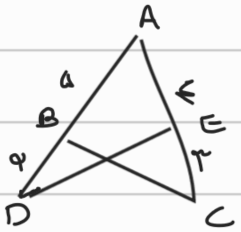
$۵۴ = ab\,\frac{۱}{۲} \rightarrow ab = ۵۴ \times ۲ = ۱۰۸$

$a = \frac{۲}{۳}b \rightarrow (\frac{۲}{۳}b)(b) = ۱۰۸ \rightarrow \frac{۲}{۳}b^۲ = ۱۰۸ \rightarrow b^۲ \times \frac{۳}{۲} \rightarrow b^۲ = ۵۴ \times ۳ \rightarrow b^۲ = ۱۶۲ \rightarrow b = ۹\sqrt{۲}$

$a = \frac{۲}{۳}(۹\sqrt{۲}) = a = ۶\sqrt{۲} \qquad P = ۲(a+b) = ۲(۶\sqrt{۲} + ۹\sqrt{۲}) = ۳۰\sqrt{۲}$

۸-

$S_{ABC} - S_{ADE} = ۱,۷۵ \qquad$ حاده $\tan\hat{A} = ?$



$S(ABC) - S(ADE) = ۱,۷۵ \qquad ۰ < A < ۹۰$

$S = \frac{۱}{۲}ab\sin\theta$

$S(ABC) = \frac{۱}{۲}(AB)(AC)\sin A = \frac{۱}{۲} \times ۵ \times ۷ \times \sin A = \frac{۳۵}{۲}\sin A = ۱۷,۵\sin A$

$S(ADE) = \frac{۱}{۲}(AD)(AE)\sin A = \frac{۱}{۲} \times ۷ \times ۴\sin A = \frac{۲۸}{۲}\sin A = ۱۴\sin A$

$۱۷,۵\sin A - ۱۴\sin A = ۱,۷۵ \rightarrow ۳,۵\sin A = ۱,۷۵ \rightarrow \sin A = \frac{۱}{۲} \rightarrow \sin A = \frac{۱,۷۵}{۳,۵} = \frac{۱}{۲}$

$\sin^2 A + \cos^2 A = 1 \rightarrow (\frac{1}{2})^2 + \cos^2 A = 1 \rightarrow \frac{1}{4} + \cos^2 = 1 \rightarrow \cos^2 A = \frac{3}{4} \rightarrow \cos\alpha = \frac{\sqrt{3}}{2}$

$\tan A = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

۹ـ

$2\overline{BN} = 3\overline{NC}$ $\quad S(\overset{\Delta}{ABC}) = 3(\overset{\Delta}{BMN})$ $\underbrace{\qquad}_{\text{مشترک } \hat{B}}$

$\Rightarrow \frac{BN}{NC} = \frac{3}{2}$

$BC = BN + NC = 3k + 2k = 5k \qquad \frac{BM}{BC} = \frac{3k}{5k} = \frac{3}{5}$

$\frac{S_{BMN}}{S_{ABC}} = \frac{BM \times BN}{BA \times BC} = \frac{1}{3} \rightarrow \frac{BN}{BC} = \frac{3}{5} \qquad \frac{1}{3} = \frac{BM}{BA} \times \frac{3}{5}$

$\frac{BM}{BA} = \frac{1}{3} \times \frac{5}{3} = \frac{5}{9}$

$\frac{BM}{BM+AM} = \frac{5}{9} \rightarrow \frac{BM}{AM} = \frac{5}{9-5} = \frac{5}{4}$

۱۰ـ

$\frac{1}{\sqrt{\cos^2\alpha}} - \tan\alpha = \frac{1+\sin\alpha}{|\cos\alpha|} \rightarrow \cos\alpha \neq 0 \rightarrow \cot\alpha \neq 0 \rightarrow \sin\alpha < 0$

$\frac{|\sin\alpha|}{\cos\alpha} = -\frac{1}{\cot\alpha}$

$\underbrace{-\frac{1}{\cot\alpha}}_{-\frac{\sin\alpha}{\cos\alpha}} \rightarrow \underbrace{|\sin\alpha| = -\sin\alpha}_{\sin\alpha \leq 0}$

$\sin\alpha \neq 0$

$\frac{-\sin\alpha}{\cos\alpha} = -\frac{\sin\alpha}{\cos\alpha} \rightarrow \sin\alpha \leq 0 \qquad \cos\alpha \neq 0 \Rightarrow$ ربع ۳ یا ۴

$\frac{1-(1+\sin\alpha)}{|\cos\alpha|} = \tan\alpha \qquad \frac{1-1-\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{\cos\alpha}$

$\frac{-1}{|\cos\alpha|} = \frac{1}{\cos\alpha} \rightarrow \frac{|\cos\alpha|}{\cos\alpha} = -1 \hookrightarrow \cos\alpha < 0$ ربع ۲و۳

$\boxed{\text{ربع سوم}}$