پونیان دهقانی

۱- الف) $\tan\frac{11\pi}{4}+\sin\frac{15\pi}{4}\cos\frac{13\pi}{4}$

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$\tan\left(2\pi+\frac{3\pi}{4}\right)+\sin\left(4\pi-\frac{\pi}{4}\right)\cos\left(2\pi+\frac{5\pi}{4}\right)=-1+\left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)=-1+\frac{1}{2}$

$=-\frac{1}{2}$

ب) $\tan\frac{17\pi}{4}\sin\frac{11\pi}{3}+\cos\frac{10\pi}{3}$

$\tan\left(4\pi-\frac{\pi}{4}\right)\sin\left(3\pi+\frac{2\pi}{3}\right)+\cos\left(3\pi+\frac{\pi}{3}\right)=-\frac{\sqrt{3}}{2}\times-\frac{\sqrt{3}}{2}+-\frac{1}{2}=\frac{3}{4}-\frac{3}{4}=0$

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۲- $\cot x=4 \rightarrow \dfrac{\frac{3\cos x}{\sin x}-\frac{\sin x}{\sin x}}{\frac{\cos x}{\sin x}+\frac{\sin x}{\sin x}}=\frac{12-1}{4+1}=\frac{11}{5}$

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۳- $\sin\alpha=2\cos\alpha$ ، ربع سوم $\rightarrow \cos\alpha=?$

۲

$\sin^2\alpha+\cos^2\alpha=1 \rightarrow (2\cos\alpha)^2+\cos^2\alpha=1 \rightarrow 4\cos^2\alpha+\cos^2\alpha=1$

$\rightarrow \cos^2\alpha=\frac{1}{5} \rightarrow \cos\alpha \overset{\text{ربع سوم}}{=} -\frac{\sqrt{5}}{5} \qquad \rightarrow 5\cos^2\alpha=1$

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۴- الف) ربع ۲ ، $\sin\alpha=\frac{\sqrt{2}}{10} \rightarrow \cos\left(\frac{11\pi}{4}+\alpha\right)=?$

$\cos\left(2\pi+\frac{3\pi}{4}\right)\ \alpha \Rightarrow \left(\frac{3\pi}{4}+\alpha\right)$

$\sin^2\alpha+\cos^2\alpha=1 \rightarrow \cancel{\left(\frac{\sqrt{2}}{10}\right)^2}+\cos^2\alpha=1$

$\frac{1}{50}$

۳

$\cos^2\alpha=\frac{50}{50}-\frac{1}{50}=\frac{49}{50} \rightarrow \cos\alpha=-\frac{7\sqrt{2}}{10}$

$\cos\left(\frac{3\pi}{4}+\alpha\right)=\cos\frac{3\pi}{4}\cos\alpha-\sin\frac{3\pi}{4}\sin\alpha=\frac{7}{10}-\frac{1}{10}=\frac{6}{10}=\frac{3}{5}$

ب) $1+\tan^2\alpha=\frac{1}{\cos^2\alpha} \rightarrow 1+\left(\frac{1}{7}\right)=\frac{1}{\cos^2} \rightarrow \frac{1}{\cos\alpha}=\frac{5\sqrt{2}}{7} \qquad \sin\left(\frac{5\pi}{4}+\alpha\right)$

$\cos\alpha=\frac{7}{5\sqrt{2}}=\frac{7\sqrt{2}}{10} \rightarrow -\frac{\sqrt{2}}{2}(\cos\alpha+\sin\alpha) \rightarrow \cos\alpha=\frac{7\sqrt{2}}{10}$

$\sin\alpha=\tan\alpha\times\cos\alpha=\frac{1}{7}\times\frac{7\sqrt{2}}{10}=\frac{\sqrt{2}}{10} \rightarrow -\frac{\sqrt{2}}{2}\left(\frac{7\sqrt{2}}{10}+\frac{\sqrt{2}}{10}\right)=-\frac{\sqrt{2}}{2}\times\frac{4\sqrt{2}}{5}=-\frac{4}{5}$

۵-

$$۲\sin^۲x+\cos^۲x=\frac{۴}{۳}\rightarrow \sin^۲x+\sin^۲x+\cos^۲x=\frac{۴}{۳}\rightarrow \sin^۲x+۱=\frac{۴}{۳}$$

۳

$$\sin^۲x=\frac{۱}{۳}\qquad \frac{۱}{۳}+\cos^۲x=۱\Rightarrow \cos^۲x=\frac{۳}{۳}-\frac{۱}{۳}=\frac{۲}{۳}\qquad \tan^۲=\frac{\sin^۲x}{\cos^۲x}=\frac{\frac{۱}{۳}}{\frac{۲}{۳}}=\frac{۱}{۲}$$

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۶-

$\sqrt{۳},\ ۶,\ \alpha \qquad S=۴,۵$

۲

$$۴,۵=\frac{۱}{۲}ab\sin\alpha\Rightarrow \frac{۱}{۲}\times\sqrt{۳}\times ۶\times\sin\alpha=۴,۵\Rightarrow ۳\sqrt{۳}\sin\alpha$$

$$\sin\alpha=\frac{۴,۵}{۳\sqrt{۳}}=\frac{۱۳,۵\sqrt{۳}}{۲۷}=\frac{\sqrt{۳}}{۲}\rightarrow ۰<\alpha<۱۸۰ \text{ یا } ۰<\alpha<\pi$$

$\underbrace{\frac{\pi}{۳}\rightarrow ۶۰}_{min}\qquad \underbrace{\frac{۲\pi}{۳}\rightarrow ۱۲۰}_{max}\qquad \frac{۱۲۰}{۶۰}=۲$ یا $\qquad \frac{\frac{۲\pi}{۳}}{\frac{\pi}{۳}}=۲$

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۷-

$S=۵۴\qquad \frac{a}{b}=\frac{۲}{۳}$

$$۵۴=ab\sin\alpha\rightarrow ۵۴=ab(\sin ۱۵۰)$$
$$\rightarrow \sin(۱۸۰-۳۰)=\sin ۳۰=\frac{۱}{۲}$$

$$۵۴=ab\,\frac{۱}{۲}\rightarrow ab=۵۴\times ۲=۱۰۸$$

$$a=\frac{۲}{۳}b\rightarrow (\frac{۲}{۳}b)(b)=۱۰۸\rightarrow \frac{۲}{۳}b^۲=۱۰۸\rightarrow b^۲\times\frac{۳}{۲}\rightarrow b^۲=۵۴\times ۳\rightarrow b^۲=۱۶۲\rightarrow b=۹\sqrt{۲}$$

$$a=\frac{۲}{۳}(۹\sqrt{۲})=a=۶\sqrt{۲}\qquad P=۲(a+b)=۲(۶\sqrt{۲}+۹\sqrt{۲})=۳۰\sqrt{۲}$$

۳

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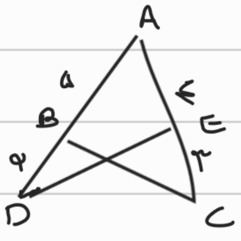
۸-

$S_{ABC}-S_{ADE}=۱,۷۵\qquad$ حاده $\tan\hat{A}=?$

۲




$S(A\hat{B}C)-S(A\hat{D}E)=۱,۷۵\qquad ۰<A<۹۰$

$$S=\frac{۱}{۲}ab\sin\theta$$

$$S(ABC)=\frac{۱}{۲}(AB)(AC)\sin A=\frac{۱}{۲}\times ۵\times ۷\times\sin A=\frac{۳۵}{۲}\sin A=۱۷,۵\sin A$$

$$S(ADE)=\frac{۱}{۲}(AD)(AE)\sin A=\frac{۱}{۲}\times ۷\times ۴\sin A=\frac{۲۸}{۲}\sin A=۱۴\sin A$$

$$۱۷,۵\sin A-۱۴\sin A=۱,۷۵\rightarrow ۳,۵\sin A=۱,۷۵\rightarrow \sin A=\frac{۱}{۲}\rightarrow \sin A=\frac{۱,۷۵}{۳,۵}=\frac{۱}{۲}$$

$\sin^۲ A + \cos^۲ A = 1 \to (\frac{1}{۲})^۲ + \cos^۲ A = 1 \to \frac{1}{۴} + \cos^۲ = 1 \to \cos^۲ A = \frac{۳}{۴} \to \cos\alpha = \frac{\sqrt{۳}}{۲}$

$\tan A = \frac{\frac{1}{۲}}{\frac{\sqrt{۳}}{۲}} = \frac{1}{\sqrt{۳}} = \frac{\sqrt{۳}}{۳}$

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۹- پ

$۲\overline{BN} = ۳\overline{NC}$ $\qquad S(\overset{\Delta}{ABC}) = ۳(\overset{\Delta}{BMN})$ مشترک $\hat{B}$

$\Rightarrow \frac{BN}{NC} = \frac{۳}{۲}$

$BC = BN + NC = ۳k + ۲k = ۵k \qquad \frac{BM}{BC} = \frac{۳k}{۵k} = \frac{۳}{۵}$

$\frac{S_{BMN}}{S_{ABC}} = \frac{BM \times BN}{BA \times BC} = \frac{1}{۳} \to \frac{BN}{BC} = \frac{۳}{۵} \qquad \frac{1}{۳} = \frac{BM}{BA} \times \frac{۳}{۵}$

$\frac{BM}{BA} = \frac{1}{۳} \times \frac{۵}{۳} = \frac{۵}{۹}$

$\frac{BM}{BM+AM} = \frac{۵}{۹} \to \frac{BM}{AM} = \frac{۵}{۹-۵} = \frac{۵}{۴}$

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۱۰- ۱

$\frac{1}{\sqrt{\cos^۲\alpha}} - \tan\alpha = \frac{1+\sin\alpha}{|\cos\alpha|} \to \cos\alpha \neq 0 \to \cot\alpha \neq 0 \to \sin\alpha < 0$

$\to \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1+\sin\alpha}{|\cos\alpha|} \to \frac{1}{|\cos\alpha|} - \frac{1+\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha}$

$\frac{|\sin\alpha|}{\cos\alpha} = -\frac{1}{\cot\alpha}$

$\underbrace{-\frac{\sin\alpha}{\cos\alpha}} \to \underbrace{|\sin\alpha| = -\sin\alpha}$

$\sin\alpha \leq 0$

$\sin\alpha \neq 0$

$-\frac{\sin\alpha}{|\cos\alpha|} = -\frac{\sin\alpha}{\cos\alpha}$

$|\cos\alpha| = -\cos\alpha$

$\cos\alpha < 0$

۳ !

$\frac{-\sin\alpha}{\cos\alpha} = -\frac{\sin\alpha}{\cos\alpha} \to \sin\alpha \geq 0 \qquad \cos\alpha \neq 0 \Rightarrow$ ~~ربع ۴~~

$\frac{1-(1+\sin\alpha)}{|\cos\alpha|} = \tan\alpha \qquad \frac{1-1-\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{\cos\alpha}$

$\frac{-1}{|\cos\alpha|} = \frac{1}{\cos\alpha} \to \frac{|\cos\alpha|}{\cos\alpha} = -1 \to \cos\alpha < 0$ ~~ربع ۲و۳~~

ربع سوم