①

الف) $\tan\frac{11\pi}{4}+\sin\frac{15\pi}{4}\cos\frac{13\pi}{4}=$

$\tan\left(\frac{8\pi}{4}+\frac{3\pi}{4}\right)+\sin\left(\frac{8\pi}{4}+\frac{7\pi}{4}\right).\cos\left(\frac{8\pi}{4}+\frac{5\pi}{4}\right)=$

$\tan(2\pi+\frac{3\pi}{4})+\sin(2\pi+\frac{7\pi}{4}).\cos(2\pi+\frac{5\pi}{4})=$

$\tan\frac{3\pi}{4}+\sin\frac{7\pi}{4}\times\cos\frac{5\pi}{4}=-1+(-\frac{\sqrt{2}}{2})(-\frac{\sqrt{2}}{2})=-1+\frac{2}{4}=-1+\frac{1}{2}=\boxed{-\frac{1}{2}}$

ب) $\tan\frac{17\pi}{6}.\sin\frac{11\pi}{3}+\cos\frac{10\pi}{3}=$

$\tan\left(\frac{12\pi}{6}+\frac{5\pi}{6}\right).\sin\left(\frac{6\pi}{3}+\frac{5\pi}{3}\right)+\cos\left(\frac{6\pi}{3}+\frac{4\pi}{3}\right)=$

$\tan(2\pi+\frac{5\pi}{6}).\sin(2\pi+\frac{5\pi}{3})+\cos(2\pi+\frac{4\pi}{3})=$

$\tan\frac{5\pi}{6}\times\sin\frac{5\pi}{3}+\cos\frac{4\pi}{3}=\tan150^\circ\times\sin300^\circ+\cos240^\circ$

$=-\frac{1}{\sqrt{3}}\times(-\frac{\sqrt{3}}{2})+(-\frac{1}{2})=+\frac{1}{2}-\frac{1}{2}=\boxed{0}$

②

$$\frac{3\cos x-\sin x}{\cos x+\sin x}=\frac{\frac{3\cos x}{\sin x}-\frac{\sin x}{\sin x}}{\frac{\cos x}{\sin x}+\frac{\sin x}{\sin x}}=\frac{3\cot x-1}{\cot x+1}=\frac{3\times4-1}{4+1}=$$

$$\frac{12-1}{4+1}=\boxed{\frac{11}{5}}$$

③

$\sin\alpha=2\cos\alpha\rightarrow\sin^2\alpha=4\cos^2\alpha$

$\sin^2\alpha+\cos^2\alpha=4\cos^2\alpha+\cos^2\alpha=5\cos^2\alpha=1$

$\cos^2\alpha=\frac{1}{5}\rightarrow\sqrt{\cos^2\alpha}=\sqrt{\frac{1}{5}}\rightarrow\cos\alpha=|\frac{1}{\sqrt{5}}|\rightarrow\cos\alpha=\pm\frac{1}{\sqrt{5}}$

$\cos\alpha=\pm\frac{\sqrt{5}}{5}\rightarrow\boxed{\cos\alpha=-\frac{\sqrt{5}}{5}}$

چون در ربع سوم دایره مثلثاتی است.

$\hookrightarrow\cos\alpha<0$

④

$\cos(\frac{11\pi}{4}+\alpha)=\cos\frac{11\pi}{4}\cos\alpha-\sin\frac{11\pi}{4}\sin\alpha$

$\cos\frac{11\pi}{4}=\cos(\frac{8\pi}{4}+\frac{3\pi}{4})=\cos(2\pi+\frac{3\pi}{4})=$

$\cos\frac{3\pi}{4}=-\frac{\sqrt{2}}{2}$

$\sin(\frac{11\pi}{4})=\sin(\frac{8\pi}{4}+\frac{3\pi}{4})=\sin(2\pi+\frac{3\pi}{4})$

$=\sin\frac{3\pi}{4}=\frac{\sqrt{2}}{2}$

$\sin^2\alpha+\cos^2\alpha=1\rightarrow\cos^2\alpha=1-\sin^2\alpha$

$=1-\frac{2}{100}=\frac{98}{100}\rightarrow\frac{\sqrt{98}}{10}\rightarrow\cos\alpha=\pm\frac{\sqrt{98}}{10}=\pm\frac{7\sqrt{2}}{10}$

$\rightarrow\cos\alpha=-\frac{7\sqrt{2}}{10}\rightsquigarrow$ چون در ربع دوم

$-\frac{\sqrt{2}}{2}\times(-\frac{7\sqrt{2}}{10})-\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{10}=\frac{7}{10}-\frac{1}{10}=\boxed{\frac{6}{10}=0.6}$

ب) $\sin(\frac{13\pi}{4}+\alpha)=\sin\frac{13\pi}{4}\cos\alpha+\cos\frac{13\pi}{4}\sin\alpha$

$\sin(\frac{13\pi}{4})=\sin(\frac{8\pi}{4}+\frac{5\pi}{4})=\sin(2\pi+\frac{5\pi}{4})=\sin\frac{5\pi}{4}=-\frac{\sqrt{2}}{2}$

$\cos(\frac{13\pi}{4})=\cos(2\pi+\frac{5\pi}{4})=\cos\frac{5\pi}{4}=-\frac{\sqrt{2}}{2}$

$1+\tan^2=\frac{1}{\cos^2\alpha}\rightarrow1+\frac{1}{49}=\frac{50}{49}\rightarrow\cos^2\alpha=\frac{49}{50}$

$\cos\alpha=\pm\frac{7}{5\sqrt{2}}=\pm\frac{7\sqrt{2}}{10}\rightarrow\cos\alpha=+\frac{7\sqrt{2}}{10}$

$\sin^2\alpha=1-\cos^2\alpha=1-\frac{49}{50}=\frac{1}{50}\rightarrow\sin\alpha=\frac{\sqrt{2}}{10}$

$-\frac{\sqrt{2}}{2}\times\frac{7\sqrt{2}}{10}+(-\frac{\sqrt{2}}{2})\times\frac{\sqrt{2}}{10}=-\frac{7}{10}-\frac{1}{10}=\boxed{-\frac{8}{10}=-0.8}$

⑥

$2\sin^2 x + \cos^2 x = \frac{4}{3}$ $\quad$ $2\sin^2 x + \cos^2 x = \frac{4}{3}$

$\sin^2 x + \cos^2 x = 1$ $\quad$ $-\sin^2 x - \cos^2 x = -1$

$\sin^2 x = \frac{1}{3}$

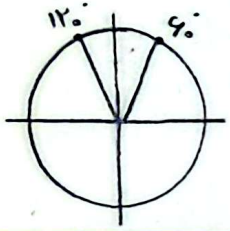
$\hookrightarrow \frac{1}{3} + \cos^2 x = 1 \rightarrow \cos^2 x = 1 - \frac{1}{3} = \frac{2}{3}$

$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \boxed{\frac{1}{2}}$

⑦

$S = \frac{1}{2} \times 9 \times \sqrt{3} \times \sin\alpha = 4,5$

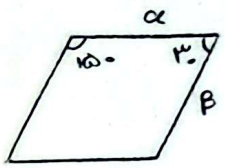
$3\sqrt{3} \times \sin\alpha = 4,5 \rightarrow \sin\alpha = \frac{4,5}{3\sqrt{3}} = \frac{1,5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1,5\sqrt{3}}{3} = \frac{\sqrt{3}}{2}$



$\hookrightarrow \alpha = 120°$ $\quad \frac{120}{60} = \boxed{2}$ برابر

$\hookrightarrow \alpha = 60°$

⑦



$\left(\frac{1}{2} \times \alpha \times \beta \times \sin 30°\right) \times 2 = \alpha \times \beta \times \frac{1}{2} = 54$

$\alpha . \beta = 108 \rightarrow \frac{2}{3}\beta \times \beta = 108$

$\frac{\alpha}{\beta} = \frac{2}{3} \rightarrow \alpha = \frac{2}{3}\beta$

$\frac{2}{3}\beta^2 = 108 \rightarrow \beta^2 = 108 \times \frac{3}{2} = 162 \rightsquigarrow \beta = \sqrt{162} = 9\sqrt{2}$

$\alpha = \frac{2}{3}\beta = \frac{2}{3} \times 9\sqrt{2} = \frac{18\sqrt{2}}{3} = 6\sqrt{2} \rightsquigarrow \alpha = 6\sqrt{2}$

$(\alpha + \beta) \times 2 = (9\sqrt{2} + 6\sqrt{2}) \times 2 = 15\sqrt{2} \times 2 = \boxed{30\sqrt{2}}$

⑧

$S_{ABC} = \frac{1}{2} \times 10 \times 7 \times \sin A = \frac{70}{2}\sin A = 17,5\sin A$

$S_{ADE} = \frac{1}{2} \times 4 \times 7 \times \sin A = 14\sin A$

$17,5\sin A - 14\sin A = 3,5\sin A = 1,75$

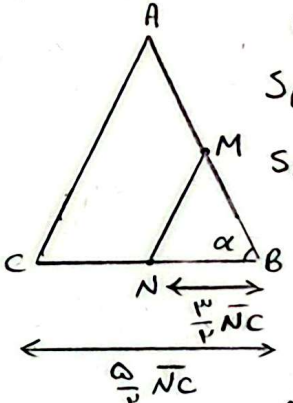
$\sin A = \frac{1,75}{3,5} = \frac{175}{350} = \frac{1}{2}$

$\sin^2 A + \cos^2 A = 1 \rightarrow \cos^2 A = 1 - \sin^2 A = 1 - \frac{1}{4} = \frac{3}{4}$

$\cos^2 A = \frac{3}{4} \rightarrow \cos A = \pm\frac{\sqrt{3}}{2} \rightarrow \cos A = \frac{\sqrt{3}}{2}$

$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \boxed{\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}}$

⑨



$2\overline{BN} = 3\overline{NC} \rightarrow \overline{BN} = \frac{3}{2}\overline{NC}$

$S_{ABC} = \frac{1}{2} \times AB \times \frac{5}{2}NC \times \sin\alpha$

$S_{MBN} = \frac{1}{2} \times MB \times \frac{3}{2}NC \times \sin\alpha$

$\frac{5}{4}NC \times AB \times \sin\alpha = \frac{9}{4}NC \times MB \times \sin\alpha$

$5AB = 9MB \rightarrow AB = \frac{9}{5}MB$

$AB = \frac{9}{5}MB \rightarrow AM + MB = \frac{9}{5}MB$

$AM = \frac{9}{5}MB - MB = \frac{4}{5}MB \rightarrow AM = \frac{4}{5}BM$

$\frac{BM}{AM} = \frac{BM}{\frac{4}{5}BM} = \boxed{\frac{5}{4} = 1,25}$

(10)

$$\frac{1}{\sqrt{\cos^2\alpha}} - \tan\alpha = \frac{1+\sin\alpha}{|\cos\alpha|}$$

$$\frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1}{|\cos\alpha|} + \frac{\sin\alpha}{|\cos\alpha|} \rightarrow -\frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{|\cos\alpha|}$$

$$\rightarrow -\frac{1}{\cos\alpha} = \frac{1}{|\cos\alpha|} \rightarrow \cos\alpha < 0.$$

$$\frac{|\sin\alpha|}{\cos\alpha} = -\frac{1}{\cot\alpha} \rightarrow \frac{|\sin\alpha|}{\cos\alpha} = -\tan\alpha \rightarrow \frac{|\sin\alpha|}{\cos\alpha} = -\frac{\sin\alpha}{\cos\alpha}$$

$$\rightarrow |\sin\alpha| = -\sin\alpha \rightarrow \sin\alpha < 0.$$

$\cos\alpha < 0$, $\sin\alpha < 0$ $\rightarrow$ ربع سوم