مهران نراقی - دهم دختر ۲ - تکلیف شماره ۱۸

۲۰ خیلی عالی حل نوشتی

۱)

الف) $\tan\frac{11\pi}{4}+\sin\frac{15\pi}{4}\cos\frac{13\pi}{4}=$

$\tan\left(\frac{8\pi}{4}+\frac{3\pi}{4}\right)+\sin\left(\frac{8\pi}{4}+\frac{7\pi}{4}\right)\cdot\cos\left(\frac{8\pi}{4}+\frac{5\pi}{4}\right)=$

$\tan\left(2\pi+\frac{3\pi}{4}\right)+\sin\left(2\pi+\frac{7\pi}{4}\right)\cdot\cos\left(2\pi+\frac{5\pi}{4}\right)=$

$\tan\frac{3\pi}{4}+\sin\frac{7\pi}{4}\times\cos\frac{5\pi}{4}=-1+\left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)=-1+\frac{2}{4}=-1+\frac{1}{2}=\boxed{-\frac{1}{2}}$

ب) $\tan\frac{17\pi}{6}\cdot\sin\frac{11\pi}{3}+\cos\frac{10\pi}{3}=$

$\tan\left(\frac{12\pi}{6}+\frac{5\pi}{6}\right)\cdot\sin\left(\frac{6\pi}{3}+\frac{5\pi}{3}\right)+\cos\left(\frac{6\pi}{3}+\frac{4\pi}{3}\right)=$

$\tan\left(2\pi+\frac{5\pi}{6}\right)\cdot\sin\left(2\pi+\frac{5\pi}{3}\right)+\cos\left(2\pi+\frac{4\pi}{3}\right)=$

$\tan\frac{5\pi}{6}\times\sin\frac{5\pi}{3}+\cos\frac{4\pi}{3}=\tan 150^\circ\times\sin 300^\circ+\cos 240^\circ$

$=-\frac{1}{\sqrt{3}}\times\left(-\frac{\sqrt{3}}{2}\right)+\left(-\frac{1}{2}\right)=+\frac{1}{2}-\frac{1}{2}=\boxed{0}$

۲)

$\frac{3\cos x-\sin x}{\cos x+\sin x}=\frac{\frac{3\cos x}{\sin x}-\frac{\sin x}{\sin x}}{\frac{\cos x}{\sin x}+\frac{\sin x}{\sin x}}=\frac{3\cot x-1}{\cot x+1}=\frac{3\times 4-1}{4+1}=$

$\frac{12-1}{4+1}=\boxed{\frac{11}{5}}$

۳)

$\sin\alpha=2\cos\alpha\rightarrow\sin^2\alpha=4\cos^2\alpha$

$\sin^2\alpha+\cos^2\alpha=4\cos^2\alpha+\cos^2\alpha=5\cos^2\alpha=1$

$\cos^2\alpha=\frac{1}{5}\rightarrow\sqrt{\cos^2\alpha}=\sqrt{\frac{1}{5}}\rightarrow\cos\alpha=\left|\frac{1}{\sqrt{5}}\right|\rightarrow\cos\alpha=\pm\frac{1}{\sqrt{5}}$

$\cos\alpha=\pm\frac{\sqrt{5}}{5}\rightarrow\boxed{\cos\alpha=-\frac{\sqrt{5}}{5}}$

چون در ربع سوم دایره مثلثاتی است.

$\rightarrow\cos\alpha<0$

۴)

$\cos\left(\frac{11\pi}{4}+\alpha\right)=\cos\frac{11\pi}{4}\cos\alpha-\sin\frac{11\pi}{4}\sin\alpha$

$\cos\frac{11\pi}{4}=\cos\left(\frac{8\pi}{4}+\frac{3\pi}{4}\right)=\cos\left(2\pi+\frac{3\pi}{4}\right)=$

$\cos\frac{3\pi}{4}=-\frac{\sqrt{2}}{2}$

$\sin\left(\frac{11\pi}{4}\right)=\sin\left(\frac{8\pi}{4}+\frac{3\pi}{4}\right)=\sin\left(2\pi+\frac{3\pi}{4}\right)$

$=\sin\frac{3\pi}{4}=\frac{\sqrt{2}}{2}$

$\sin^2\alpha+\cos^2\alpha=1\rightarrow\cos^2\alpha=1-\sin^2\alpha$

$=1-\frac{2}{100}=\frac{98}{100}\rightarrow\frac{\sqrt{98}}{10}\rightarrow\cos\alpha=\pm\frac{\sqrt{98}}{10}=\pm\frac{7\sqrt{2}}{10}$

$\rightarrow\cos\alpha=-\frac{7\sqrt{2}}{10}\rightsquigarrow$ چون در ربع دوم

$-\frac{\sqrt{2}}{2}\times\left(-\frac{7\sqrt{2}}{10}\right)-\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{10}=\frac{7}{10}-\frac{1}{10}=\boxed{\frac{6}{10}=0/6}$

ب) $\sin\left(\frac{13\pi}{4}+\alpha\right)=\sin\frac{13\pi}{4}\cos\alpha+\cos\frac{13\pi}{4}\sin\alpha$

$\sin\left(\frac{13\pi}{4}\right)=\sin\left(\frac{8\pi}{4}+\frac{5\pi}{4}\right)=\sin\left(2\pi+\frac{5\pi}{4}\right)=\sin\frac{5\pi}{4}=-\frac{\sqrt{2}}{2}$

$\cos\left(\frac{13\pi}{4}\right)=\cos\left(2\pi+\frac{5\pi}{4}\right)=\cos\frac{5\pi}{4}=-\frac{\sqrt{2}}{2}$

$1+\tan^2=\frac{1}{\cos^2\alpha}\rightarrow 1+\frac{1}{49}=\frac{50}{49}\rightarrow\cos^2\alpha=\frac{49}{50}$

$\cos a=\pm\frac{7}{5\sqrt{2}}=\pm\frac{7\sqrt{2}}{10}\rightarrow\cos a=+\frac{7\sqrt{2}}{10}$

$\sin^2\alpha=1-\cos^2\alpha=1-\frac{49}{50}=\frac{1}{50}\rightarrow\sin\alpha=\frac{\sqrt{2}}{10}$

$-\frac{\sqrt{2}}{2}\times\frac{7\sqrt{2}}{10}+\left(-\frac{\sqrt{2}}{2}\right)\times\frac{\sqrt{2}}{10}=-\frac{7}{10}-\frac{1}{10}=\boxed{-\frac{8}{10}=-0/8}$

۵

$$2\sin^2x+\cos^2x=\frac{4}{3}$$
$$\sin^2x+\cos^2x=1$$

$$\begin{array}{r} 2\sin^2x+\cos^2x=\frac{4}{3} \\ -\sin^2x-\cos^2x=-1 \\ \hline \sin^2x=\frac{1}{3} \end{array}$$

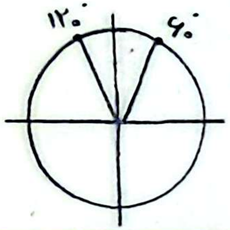
$$\frac{1}{3}+\cos^2x=1 \rightarrow \cos^2x=1-\frac{1}{3}=\frac{2}{3}$$

$$\tan^2x=\frac{\sin^2x}{\cos^2x}=\frac{\frac{1}{3}}{\frac{2}{3}}=\boxed{\frac{1}{2}}$$

۶

$$S=\frac{1}{2}\times 6\times\sqrt{3}\times\sin\alpha=4.5$$

$$3\sqrt{3}\times\sin\alpha=4.5\rightarrow\sin\alpha=\frac{4.5}{3\sqrt{3}}=\frac{1.5}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{1.5\sqrt{3}}{3}=\frac{\sqrt{3}}{2}$$



$\alpha=120°$

$\alpha=60°$

$\frac{120°}{60°}=\boxed{2}$ برابر

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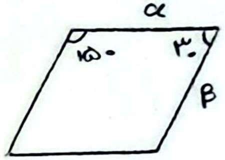


$$\left(\frac{1}{2}\times\alpha\times\beta\times\sin 30°\right)\times 2=\alpha\times\beta\times\frac{1}{2}=54$$

$$\alpha.\beta=108\rightarrow\frac{2}{3}\beta\times\beta=108$$

$$\frac{\alpha}{\beta}=\frac{2}{3}\rightarrow\alpha=\frac{2}{3}\beta$$

$$\frac{2}{3}\beta^2=108\rightarrow\beta^2=108\times\frac{3}{2}=162\leadsto\beta=\sqrt{162}=9\sqrt{2}$$

$$\alpha=\frac{2}{3}\beta=\frac{2}{3}\times 9\sqrt{2}=\frac{18\sqrt{2}}{3}=6\sqrt{2}\leadsto\alpha=6\sqrt{2}$$

$$(\alpha+\beta)\times 2=(9\sqrt{2}+6\sqrt{2})\times 2=15\sqrt{2}\times 2=\boxed{30\sqrt{2}}$$

۸

$$S_{ABC}=\frac{1}{2}\times 5\times 7\times\sin A=\frac{35}{2}\sin A=17.5\sin A$$

$$S_{ADE}=\frac{1}{2}\times 4\times 7\times\sin A=14\sin A$$

$$17.5\sin A-14\sin A=3.5\sin A=1.75$$

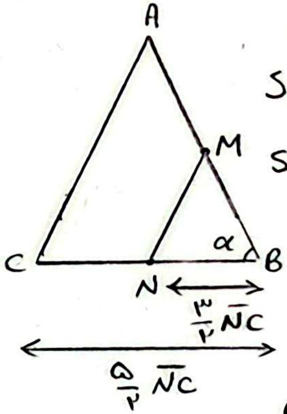
$$\sin A=\frac{1.75}{3.5}=\frac{175}{350}=\frac{1}{2}$$

$$\sin^2A+\cos^2A=1\rightarrow\cos^2A=1-\sin^2A=1-\frac{1}{4}=\frac{3}{4}$$

$$\cos^2A=\frac{3}{4}\rightarrow\cos A=\pm\frac{\sqrt{3}}{2}\rightarrow\cos A=\frac{\sqrt{3}}{2}$$

$$\tan A=\frac{\sin A}{\cos A}=\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}=\boxed{\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}}$$

۹



$$2\overline{BN}=3\overline{NC}\rightarrow\overline{BN}=\frac{3}{2}\overline{NC}$$

$$S_{ABC}=\frac{1}{2}\times AB\times\frac{5}{2}NC\times\sin\alpha$$

$$S_{MBN}=\frac{1}{2}\times MB\times\frac{3}{2}NC\times\sin\alpha$$

$$\frac{5}{\cancel{4}}\cancel{NC}\times AB\times\cancel{\sin\alpha}=\frac{9}{\cancel{4}}\cancel{NC}\times MB\times\cancel{\sin\alpha}$$

$$5AB=9MB\rightarrow AB=\frac{9}{5}MB$$

$$AB=\frac{9}{5}MB\rightarrow AM+MB=\frac{9}{5}MB$$

$$AM=\frac{9}{5}MB-MB=\frac{4}{5}MB\rightarrow AM=\frac{4}{5}BM$$

$$\frac{BM}{AM}=\frac{BM}{\frac{4}{5}BM}=\boxed{\frac{5}{4}=1.25}$$

Scanned with CamScanner

۱۰

$$\frac{1}{\sqrt{\cos^2\alpha}} - \tan\alpha = \frac{1+\sin\alpha}{|\cos\alpha|}$$

$$\frac{1}{\cancel{|\cos\alpha|}} - \frac{\sin\alpha}{\cos\alpha} = \frac{1}{\cancel{|\cos\alpha|}} + \frac{\sin\alpha}{|\cos\alpha|} \to -\frac{\cancel{\sin\alpha}}{\cos\alpha} = \frac{\cancel{\sin\alpha}}{|\cos\alpha|}$$

$$\to -\frac{1}{\cos\alpha} = \frac{1}{|\cos\alpha|} \to \cos\alpha < 0$$

$$\frac{|\sin\alpha|}{\cos\alpha} = -\frac{1}{\cot\alpha} \to \frac{|\sin\alpha|}{\cos\alpha} = -\tan\alpha \to \frac{|\sin\alpha|}{\cancel{\cos\alpha}} = -\frac{\sin\alpha}{\cancel{\cos\alpha}}$$

$$\to |\sin\alpha| = -\sin\alpha \to \sin\alpha < 0$$

$\cos\alpha < 0 \ , \ \sin\alpha < 0 \to$ ربع سوم