① $1 / 2, 3, 4 / 5, 6, 7, 8, 9$

$8^2 = 64 \quad 9^2 = 81 \longrightarrow \frac{6 \times 5 + 11}{2} = \boxed{73}$

② $\{1, 2, 3\} \quad \{4, 5, 6, 7, 8, 9, 10, 11, 12\} \quad \{13, 14, 15, \ldots, 39\}$

$\{40, 41, 42, \ldots, 120\} \quad \{121, \ldots, 363\} \quad \frac{363 + 121}{2} = \boxed{242}$

③ $n = 3k \rightarrow n = 3, 6, 9, 12, \ldots, 117 \quad k = 1, 2, 3, 4, \ldots$

$n = 3k+1 \rightarrow n = 1, 4, 7, 10, \ldots \quad k = 0, 1, 2, 3, \ldots$

$n = 3k+2 \rightarrow n = 2, 5, 8, 11, \ldots \quad k = 0, 1, 2, 3, \ldots$

$n = 1 \rightarrow k = 0 \rightarrow a_1 = 4 \quad / \quad n = 2 \rightarrow k = 0 \rightarrow a_2 = \left[\frac{2}{2}\right] + a = 1 + a \quad / \quad n = 3 \rightarrow k = 1 \rightarrow a_3 = 2 + 1 = 3$

$n = 4 \rightarrow k = 1 \rightarrow a_4 = 2 \quad / \quad n = 5 \rightarrow k = 1 \rightarrow a_5 = \left[\frac{3}{2}\right] + a = 1 + a \quad / \quad n = 6 \rightarrow k = 2 \rightarrow a_6 = 4$

$n = 7 \rightarrow k = 2 \rightarrow a_7 = 2 \quad / \quad n = 8 \rightarrow k = 2 \rightarrow a_8 = \left[\frac{8}{4}\right] + a = 2 + a \quad / \quad n = 9 \rightarrow k = 3 \rightarrow a_9 = 8$

* اولین صفر

④ $a_3 = 14, a_7 = 17/2 \quad a = \frac{1}{10} \times -14 = -\frac{1}{5} \rightarrow \boxed{-\frac{1}{5}n^2 + 4n - 1}$

$a_3 = -\frac{1}{5} \times 25 + 3b + c = 14 \rightarrow 3b + c = 19$

$a_7 = -\frac{1}{5} \times 49 + 7b + c = 17/2 \rightarrow 7b + c = 27 \Rightarrow 4b = 8 \Rightarrow b = 4, c = -1$

$\frac{a_{15}}{a_1} = \frac{-\frac{1}{5} \times 15^2 + 15 \times (4) - 1}{-\frac{1}{5} + 4 - 1} = \frac{14}{\frac{14}{5}} = \boxed{5}$

⑤ $\text{جمله عمومی} = an + b \rightarrow L_{10} = 10a + b = 0 \rightarrow b = -10a$

$L_n = an - 10a \rightarrow a(n-10)$

$A_4 = L_2 \rightarrow a(2-10) = -8a \quad A_8 = L_7 = a(7-10) = -3a$

$A_8 - A_4 = 4d \rightarrow 4d = 5a \rightarrow d = \frac{5}{4}a \quad A_{15} = \frac{a(15-10)}{\frac{5}{4}a} = \frac{5a}{\frac{5}{4}a} = \boxed{4}$

⑥ $4(a_1 + d)^2 = 5(a_1 + 2d)a_1 + 3(a_1 + d)a_1$

$2a_1^2 + a_1 d - 4d^2 = 0 \quad \frac{a_4}{d} \rightarrow \frac{a_1 + 3d}{d} = \frac{a_1}{d} + 3$

$\div d^2 \leftarrow 2\left(\frac{a_1}{d}\right)^2 + \frac{a_1 d}{d^2} - \frac{4d^2}{d^2} = 0 \rightarrow 2\left(\frac{a_1}{d}\right)^2 + \frac{a_1}{d} - 4 = 0 \rightarrow \Delta = 1 + 32 = 33 \rightarrow \sqrt{\Delta} = 7$

$\frac{-1 \pm 7}{4} = \begin{cases} -2 \\ \frac{11}{8} \end{cases}$

$\frac{a_4}{d} = \frac{a_1}{d} + 3 = \begin{cases} 1 \\ \frac{11}{8} \end{cases}$

$2b = a + c$ * $\frac{a^2}{4} = \frac{cb}{4} \rightarrow a^2 = cb$ ⑦

$q = \frac{\frac{a}{2}}{\frac{c}{4}} = \frac{a}{2} \times \frac{4}{c} = \frac{2a}{c}$, $q = \frac{b}{\frac{a}{2}} = \frac{2b}{a} \rightarrow \frac{2b}{a} = \frac{2a}{c} \Rightarrow \frac{a}{c} = \frac{b}{a}$

$c = 2b - a = \frac{a^2}{b} \longrightarrow a^2 = (2b - a)b \rightarrow a^2 = 2b^2 - ab \rightarrow a^2 - 2b^2 + ab = 0$ ← $\div b^2$

$(\frac{a}{b} + 2)(\frac{a}{b} - 1) = 0 \Leftarrow (\frac{a}{b})^2 + (\frac{a}{b}) - 2 = 0$

$\frac{a}{b} = \begin{cases} -2 \\ 1 \end{cases}$ ↙ $\Rightarrow a = b$ یا $a = -2b$

$* : 2b = -2b + c \rightarrow 4b = c$

$q = \frac{4b}{4b} = 1$ $2q = \boxed{-2}$

⑧

$b = aq$, $c = aq^2$, $4a = 3b + c \rightarrow 4a = aq^2 + 3aq \xrightarrow{\div a} 4 = q^2 + 3q$

$q^2 + 3q - 4 = 0 \rightarrow (q + 4)(q - 1) = 0 \Rightarrow q = -4$ $\frac{a_9}{a_4} = q^3 = \boxed{-64}$

⑨

$\frac{a_1 q^{10}}{a_1^3 q^2} + \frac{aq}{a^2} = \frac{q^2}{a_1^2} + \frac{q}{a_1} = 2 \rightarrow \frac{q^2}{a_1^2} + \frac{q}{a_1} - 2 = 0$

$(\frac{q}{a_1} + 2)(\frac{q}{a_1} - 1) = 0 \rightarrow \frac{q}{a_1} = 1 \text{ یا } -2$

$\frac{a^2}{aq} = \frac{a}{q} = \boxed{1 \text{ یا } -\frac{1}{2}}$

⑩

$a_4 = \sqrt{64}$, $a_5 = 27$

$a_2^2 = a_2 q \Rightarrow a_2 = q$ $\frac{a_5}{a_2} = q^2 = \frac{27}{q} = q^2 \Rightarrow 27 = q^3 \Rightarrow q = 3$

$a_1 = \frac{a_5}{q^4} \rightarrow \frac{27}{81} = \frac{1}{3}$ $\frac{1}{2} - \frac{1}{3} = \boxed{\frac{1}{6}}$

ادامه ③: $a_1 + a_2 + \dots + a_9 = 1 + 4 + 1 + a + 2 + 2 + 1 + a + 4 + 0 + 2 + 2a + 1 + 8 = 23 + 3a = 9$

$a_2 + a_3 + a_8 + \dots + a_{29} = 1 + a + 1 + a + 8(2 + a) = 18 + 10a = \boxed{-2}$ ↳ $a = -2$