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نام و نام خانوادگی ...نرگس اسلامی نژاد... **پاسخنامه تشریحی تکلیف شماره** ..۱۷... **کلاس** ..پنجشنبه دختر ع......

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$\{1\}, \{2,3,4\}, \{5,6,7,8,9\}, \{10, \dots\}$

عدد آخر $\to 1, 4, 9, 16 \to n^2 \to 9^2 = \boxed{81}$

عدد اول $\to 1, 2, 5, 10$ (اختلاف‌ها: $1, 3, 5$؛ $a = \frac{2}{2}$) $\quad a=1,\ c=2 \quad an^2+bn+c$

$n^2+bn+2 \xrightarrow{n=1} 1+b+2=1 \to \boxed{b=-2} \qquad n^2-2n+2$

$n=9 \to 9^2 - 2\times 9 + 2 = 81-18+2 = \boxed{65}$

$65, \square, 81 \to \frac{81+65}{2} = \boxed{73}$

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$(1,2,3)\ (4,5,6,7,8,9,10,11,12)$

شروع دسته ۳ $\to 13$ — پایان دسته ۳ $\to 39$

شروع دسته ۴ $\to 40$ — پایان دسته ۴ $\to 120$

شروع دسته ۵ $\to 121$ — پایان دسته ۵ $\to 363$ — $\frac{363+121}{2} = \boxed{242}$

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$2^k \ ;\ n=3k \to n = 0,3,6,9,12,\dots$ (داریم $a_0=1$)

$-2k+4 \ ;\ n=3k+1 \to n=1,4,7,10,\dots$

$\left[\frac{n}{k+2}\right]+a \ ;\ n=3k+2 \to n=2,5,8,11,\dots$

$a_0+a_1+\dots+a_9=10$

$a_0=1 \quad a_3=2 \quad a_6=4 \quad a_9=8$

$a_1=4 \quad a_4=2 \quad a_7=0$

$a_2=1+a \quad a_5=1+a \quad a_8=2+a$ $\Rightarrow 25+3a=19 \to \boxed{a=-2}$

$a_2+a_5+a_8+\dots+a_{29}$

$2, 5, 8, 11, 14, 17, 20, 23, 26, 29$

$1+a, 1+a, 2+a, 2+a, 2+a, 2+a, 3+a, 3+a, 3+a, 4+a \to 10\overset{-2}{a}(2+16) = -20+18 = \boxed{-2}$

می‌دانیم:
$n=11 \to k=3$
$n=14 \to k=4$
$n=17 \to k=5$
$n=20 \to k=6$
$n=29 \to k=9$

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$a_n = an^2+bn+c \Rightarrow a_n = An^2+Bn+C$

$A = \frac{1}{70}\times -14 = -\frac{14}{70} = -\frac{1}{5}$

$a_5 \to -\frac{1}{5}\times 25 + 5B + C = 14 \quad\to\quad 5B+C=19$

$a_7 \to -\frac{1}{5}\times 49 + 7B + C = 17.2 \quad\to\quad 7B+C=27$

$2B = 8 \to \boxed{B=4} \,/\, \boxed{C=-1}$

$a_n = -\frac{1}{5}n^2+4n-1 \to a_{15} = -\frac{1}{5}\times 15^2 + 4\times 15 - 1 = 14 \to \frac{a_{15}}{a_1} = \frac{14}{\frac{14}{5}} = \frac{5\times 14}{14} = \boxed{5}$

$\to a_1 = -\frac{1}{5}\times 1 + 4\times 1 - 1 = \frac{14}{5}$

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اولی $\to a, d$ ؛ دومی $\to a', d'$

$a_4 = a'_2 \to a_1+3d = a'_1+d'$

$a_8 = a'_7 \to a_1+7d = a'_1+6d'$

دومی $\to a'_{10} = 0 \to a'_1+9d' = 0 \to a'_1 = -9d'$

$a+3d = -8d'$

$a+7d = -3d'$

$4d = 5d' \longrightarrow d' = \frac{4}{5}d$

$a'_{15} = a'_1 + 14d' \to a'_{15} = 5d' = 5\times\frac{4}{5}d = 4d$ (که در آن $a'_1 = -9d'$)

۴ برابر

$6a_2^2 = 5a_3a + 3a_2a \Rightarrow 6(a+d)^2 = 5(a+2d)a + 3(a+d)a$ $6(a^2+d^2+2ad) = 5a^2 + 10da + 3a^2 + 3ad \Rightarrow 6a^2 + 12ad + 6d^2 = 8a^2 + 13ad$ $\Rightarrow 8a^2 + 13ad - 6a^2 - 12ad - 6d^2 = 0 \Rightarrow 2a^2 + ad - 6d^2 = 0 \xrightarrow{\div d^2} 2(\frac{a}{d})^2 + \frac{a}{d} - 6 = 0$ $\frac{a}{d} = x \Rightarrow 2x^2 + x - 6 = 0 \to \Delta = b^2 - 4ac = 1 + 48 = 49 \to \sqrt{\Delta} = 7$ $x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-1 \pm 7}{4} = x = \frac{6}{4}, -2 \Rightarrow \frac{a}{d} = \frac{3}{2}, -2$ $\frac{a_4}{d} = \frac{a_1 + 3d}{d} = \frac{a}{d} + 3 \Rightarrow \frac{3}{2} + 3 = \frac{9}{2} \quad -2 + 3 = 1$	۶
$b = \frac{a+c}{2} \to {}^{*}2b = a + c$ $\frac{c}{4} = \frac{ra}{2} \to c = \frac{4 \times ra}{2} = 2ra$ $2(\frac{a}{2} = rb) \to a = 2rb \to b = \frac{a}{2r}$ $^{*}\, 2(\frac{a}{2r}) = a + 2ra \to \frac{a}{r} = a + 2ra \to (\frac{1}{r} = 1 + 2r) \to 1 = r + 2r^2 \to 2r^2 + r - 1 = 0 \to \Delta = b^2 - 4ac = 9$ $r = \frac{-1 \pm \sqrt{9}}{4} = \frac{-1 \pm 3}{4} \to r = \frac{1}{2}$ ✗ $, -1 \xrightarrow{\times 2} -2$ $a = b \Rightarrow$ پس غیرقابل قبول	۷
$b = ar$, $c = ar^2$; $2a = \frac{3b + c}{2}$ $2a = \frac{3ar + ar^2}{2} \to 2 = \frac{3r + r^2}{2} \to 4 = 3r + r^2 \to r^2 + 3r - 4 = 0 \to (r+4)(r-1) = 0$ $r = -4 \ / \ r = 1$ ✗ → چون نامتمایز $\frac{a_9}{a_6} = \frac{ar^8}{ar^5} = r^3 \to (-4)^3 = -64$	۸
$\frac{a_6}{(a_2)^3} + \frac{a_2}{(a)^2} = 2 \to \frac{a_1r^5}{(a_1r)^3} + \frac{ar}{a^2} = 2 \to \frac{a_1r^5}{a_1^3r^3} + \frac{ar}{a^2} = 2$ $\frac{r^2}{a^2} + \frac{r}{a} = 2 \xrightarrow{\frac{r}{a} = x} x^2 + x = 2 \to x^2 + x - 2 = 0 \to (x+2)(x-1) = 0 \to x = -2, x = 1$ $\frac{r}{a} = -2 \ / \ \frac{r}{a} = 1 \to \frac{a^2}{ar} = \frac{a}{r} \to 1, -\frac{1}{2}$	۹
$a_3 = ar^2$, $a_4 = ar^3$ $\quad a_3 = \sqrt{a_4} \to (ar^2 = \sqrt{ar^3})^2 \to a^2r^4 = ar^3 \to ar = 1 \to a = \frac{1}{r}$ $a_5 = ar^4 \to \frac{1}{r} \times r^4 = r^3 = 27 \to r = 3 \ / \ a_1 = \frac{1}{3}$ $\frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$	۱۰