

۱

$$\left\{ \begin{matrix} 1 \\ 1 \end{matrix} \right\}, \left\{ \begin{matrix} 2, 3 \\ 2 \end{matrix} \right\}, \left\{ \begin{matrix} 4, 5, 6, 7, 8, 9 \\ 4 \end{matrix} \right\}$$

↓ ↓ ↓ ↓ ↓

کلاس: ۱ ۳ ۵ ۳۲ → تعداد در هر کلاس = ۲n-1

عدد آخر = ۹۲ = ۸۱ → عدد اول = ۸۱ - ۱۴ = ۴۵

کلاس = ۲ × ۵ - ۱ = ۹ → عدد اول = ۸۱ - ۱۴ = ۴۵

و اما میانگین = $\frac{\text{عدد آخر} + \text{عدد اول}}{(n+1)}$ = $\frac{۴۵ + ۸۱}{۲(۸۱+۹)}$ = $\frac{۱۲۶}{۱۸۰}$ = $\frac{۷}{۱۰}$

۲

$$\{1, 2, 3\}, \{4, 5, \dots, 11, 12\}, \{13, \dots, 34\}, \{35, \dots, 100\}, \{101, \dots, 343\}$$

تعداد = $\frac{343-101}{1} + 1 = 243$

مجموع = $\frac{(101+343)}{2} \times 243 = 511 \cdot 4$

میانگین = $\frac{511 \cdot 4}{243} = 844$

۳

$$\begin{cases} rk \rightarrow n = rk \\ -rk + k \rightarrow n = rk + 1 \\ \left\lfloor \frac{n}{k+r} \right\rfloor + a \rightarrow n = rk + r \end{cases} \rightarrow \begin{cases} n = 0, 3, 4, 9, 12, \dots \\ n = 1, 4, 5, 10, 13, \dots \\ n = 2, 6, 8, 11, 14, \dots \end{cases}$$

$$\begin{cases} a_0 = 1, a_1 = 4, a_2 = 1+a, a_3 = 2 \\ a_4 = 2, a_5 = 1+a, a_6 = 3, a_7 = 0 \\ a_n = 2+a, a_n = 1, a_{n+1} = -2 \end{cases}$$

مجموع = $1 + 4 + 1 + 2 + 1 + a + 2 + 0 + 2 + a + 1 + 1 = 19 \rightarrow 3a = -4 \rightarrow a = -\frac{4}{3}$

$a_2 + a_5 + a_8 + \dots + a_{10} = 2 \cdot (a + (1+2+\dots+2)) = 2 \cdot (a + 11) = -2$

۴

$$t_n = an^2 + bn + c \rightarrow a = \frac{1}{v_0} (-t_0) \Rightarrow \frac{1}{v_0} \times -14 = \frac{1}{\omega} = a$$

$$\begin{cases} t_0 = -\frac{1}{\omega} \omega^2 + b\omega + c = 14 \rightarrow -\omega + b\omega + c = 14 \rightarrow b\omega + c = 19 \\ t_7 = -\frac{1}{\omega} (7)^2 + 7b + c = 1712 \rightarrow -49 + 7b + c = 1712 \rightarrow 7b + c = 1761 \end{cases} \rightarrow \underline{b = 250} \rightarrow \underline{c = -1}$$

$t_n = -\frac{1}{\omega} n^2 + 250n - 1 \rightarrow t_{10} = -\frac{1}{\omega} (10)^2 + 250 \times 10 - 1 = -100 + 2500 - 1 = 2399 = t_{10}$

$t_1 = -\frac{1}{\omega} (1)^2 + 250 \times 1 - 1 = \frac{14}{\omega}$

$\frac{t_{10}}{t_1} = \frac{2399}{\frac{14}{\omega}} = \omega$

۵

$$a_k = a + kd, a_n = a + nd$$

$$b_k = b + kd', b_n = b + nd'$$

$kd = nd' \rightarrow k = \frac{nd'}{d}$

$b_{10} = b_1 + 9d'$

$\frac{b_{10}}{d} = \frac{b_1 + 9d'}{d} = \frac{nd'}{d} = k$

$$r(a+r)^r = \Delta a^r + r a^r a \Rightarrow r(a+d)^r = \omega(a+rd)(a) + r(a+d)(a)$$

$$\Rightarrow r a^r + r d^r + r a d = \Delta a^r + \Delta a d + r a^r + r a d \Rightarrow r a^r + r d^r + r a d = \Delta a^r + \Delta a d \Rightarrow r a^r + r a d - \Delta a^r = 0$$

$$\Delta = d^r - r(r)(-rd^r) = d^r + r^2 d^r = r^2 d^r, \quad a = \frac{d \pm \sqrt{r^2 d^r}}{r} \begin{cases} a = -rd \\ a = \frac{r}{r} d \end{cases}$$

$$\frac{a^r}{d} = \frac{a+rd}{d} \begin{cases} \frac{-rd+rd}{d} = \underline{1} \\ \frac{\frac{r}{r}d+rd}{d} = \underline{\frac{r}{r}+1} \end{cases}$$

$$a, b, c \Rightarrow b-d, b, b+d$$

$$b, \frac{a}{r}, \frac{c}{r} \Rightarrow b, \frac{b-d}{r}, \frac{b+d}{r} \Rightarrow \left(\frac{b-d}{r}\right)^r = b \cdot \frac{b+d}{r} \Rightarrow \frac{b^r + d^r - rbd}{r} = \frac{b^r + bd}{r} \dots$$

$$\Rightarrow \frac{d^r}{r} - \frac{rbd}{r} = 0 \Rightarrow d^r = rbd \begin{cases} d = 0 \\ d = rb \end{cases} \Rightarrow b / -\frac{rb}{r} / \frac{rb}{r} \Rightarrow b, -b, b$$

$$q = -1 \rightarrow r \times (-1) = \underline{-r}$$

$$a, b, c \Rightarrow \frac{a}{q}, a, aq \rightarrow \frac{r a}{q} = \frac{r a + a q}{r} \Rightarrow r a = r a q + a q^r \rightarrow q^r + r q - r = 0$$

$$r b, r a, c \Rightarrow r a, \frac{r a}{q}, \frac{a q}{r}$$

$$\Delta = b^r - r a c = a + 14 = r a \rightarrow z = \frac{-b \pm \sqrt{r a}}{r a} \begin{cases} \frac{r}{r} = 1 \\ -\frac{1}{r} = -r \end{cases}$$

$$\frac{a q^r}{(a q)^r} = \frac{a}{q} \Rightarrow \frac{a a}{a q} = \frac{a q^r}{a q^r} = q^r \begin{cases} q^r = 1 \rightarrow q^r = \underline{1} \\ q = -r \rightarrow q^r = \underline{-r^r} \end{cases}$$

$$\frac{a q^r}{(a q)^r} + \frac{a q}{a^r} = r \Rightarrow \frac{a q^r}{a^r a^r} + \frac{a q}{a^r} = r \Rightarrow \frac{q^r + a q}{a^r} = r \Rightarrow r a^r = q^r + a q \Rightarrow q^r + a q - r a^r = 0$$

$$\Rightarrow \Delta = b^r - r a c \Rightarrow a^r + r a^r = a a^r \Rightarrow q = \frac{-a \pm \sqrt{r a^r}}{r} \begin{cases} q = -r a \\ q = a \end{cases}$$

$$\frac{a^r}{a^r} = \frac{a^r}{a q} \begin{cases} q = -r a \rightarrow \frac{a^r}{-r a^r} = \underline{-\frac{1}{r}} \\ q = a \rightarrow \frac{a^r}{a^r} = \underline{1} \end{cases}$$

$$a_r = \sqrt{a r} = (a q^r = \sqrt{a q^r})^r \Rightarrow a^r q^r = a q^r \rightarrow a q > 1 \rightarrow a = \frac{1}{q}$$

$$a_d = r v \Rightarrow a q^r = a \cdot q \cdot q^r = r v \Rightarrow q^r = r v \rightarrow q = r$$

$$\rightarrow a = \frac{1}{r} \Rightarrow \frac{1}{r} - \frac{1}{r} = \underline{\frac{1}{r}}$$