باران آقاجانی — تکلیف ۱۷ : الگو (فصل ۳)

(۱)

$[1], [2,3,4], [5,6,7,8,9], \ldots$; $1, 3, 5, \ldots$

آخرین عدد $= 9^2 = 81$ ، دسته ۹ ام

تعداد $= \frac{81-n}{1} + 1 = 17$

تعداد اعداد دسته $= (2\times 9)-1 = 17$

$n = 65$

وسط جمله $= \frac{81+65}{2} = 73$

(۲)

$[1,\ldots,3],[4,\ldots,12][13,\ldots,39],[40,\ldots,120],[121,\ldots,363]$

$\frac{121+363}{2} = 242$

(۳)

$a_0 + a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9$

$3^k = n = 3k \Rightarrow n = 0, 3, 6, 9 \to 3^0 = 1,\ 3^1 = 3,\ 3^2 = 9,\ 3^3 = 27$ -

$\left[\frac{n}{k+2}\right] + a \to \left[\frac{3}{0+2}\right] + a,\ \left[\frac{5}{1+2}\right] + a,\ \left[\frac{8}{2+2}\right] + a \Rightarrow 1+a+1+a+2+a$

$S_{10} = 1+2+4+8+2+2+0+1+a+1+a+2+a = 19 \to 25 + 3a = 19 \Rightarrow a = -2$

$a_2 + a_5 + \cdots + a_{29} \Rightarrow n = 3k+2 \Rightarrow \left[\frac{2}{0+2}\right] - 2, \left[\frac{5}{1+2}\right] - 2, \ldots + \left[\frac{29}{9+2}\right] - 2 \Rightarrow$

$1-2+1-2+\ldots(2-2) = -2$

(۴)

$49a + 7b + c = 17/2$

$-25a + 5b + c = 14$

$24a + 2b = 3/2 \to -\frac{24}{5} + 2b = \frac{32}{10}$ $\quad 2b = 8 \quad b = 4$

$a = \frac{1}{70} \times -14 = -\frac{1}{5} = a$

$(\underbrace{25 \times -\frac{1}{5}}_{-5}) + (\underbrace{5\times 4}_{20}) + c = 14 \Rightarrow c = -1$

$a_{15} = (\underbrace{225 \times -\frac{1}{5}}_{-45})(\underbrace{15\times 4}_{60}) - 1 = 14$

$\frac{a_{15}}{a_1} = \frac{\frac{14}{1}}{\frac{14}{5}} = 5$ جواب

$a_1 = (1\times -\frac{1}{5})(1\times 4) - 1 = \frac{14}{5}$ ← $-\frac{1}{5} + \frac{20}{5} - \frac{5}{5}$

(۵)

$a_4 = 3d + a = a'_4 = 4a' + b$

$a_8 = 7d + a = a'_8 = 8a' + b$

$a'_{10} = 0 \Rightarrow 10a' + b = 0 \quad b = -10a'$

$a'_4 = 4a' - 10a' = -6a' \Rightarrow$

$a_8 = 8a' - 10a' = -2a'$

$-3d + a = +6a'$

$7d + a = -2a'$

$4d = 8a'$

قدر نسبت $\leftarrow d = \frac{8}{4}a'$

$a'_{10} = 10a' - 10a'$

$8a'$

$\frac{\frac{8a'}{1}}{\frac{8a'}{4}} = 4$

$9(a^2+d^2+2ad) = 5(a^2+2da) + 3(a^2+da) \Rightarrow 9a^2+9d^2+18ad = 8a^2+13da$

$2a^2 + da - 6d^2 \Rightarrow a^2 + \frac{1}{2}da - 3d^2 = 0$

$(a+2d)(a-\frac{3}{2}d) \Rightarrow$

$a = \begin{cases} -2d \\ +\frac{3}{2}d \end{cases}$ $\quad a_4 = a + 3d \begin{cases} -2d + 3d = d \\ +\frac{3}{2}d + \frac{6}{2}d = \frac{9}{2}d = 4.5d \end{cases}$ $\quad \frac{a_4}{a} \begin{cases} \frac{d}{d} = 1 \\ \frac{4.5d}{d} = 4.5 \end{cases}$

$a, b, c \qquad b, \frac{a}{2}, \frac{c}{4} \Rightarrow$

$a \quad a+d \quad a+2d \qquad a+d \quad \frac{a}{2} \quad \frac{a+2d}{4}$

$\frac{a^2}{4} = (a+d)\left(\frac{a+2d}{4}\right) \Rightarrow \cancel{\frac{a^2}{4}} = \cancel{\frac{a^2}{4}} + \frac{3da}{4} + \frac{2d^2}{4} \Rightarrow$

$\frac{3da}{4} + \frac{2d^2}{4} = 0 \qquad \frac{1}{4}d(3a+2d) = 0 \qquad d \neq 0 \Rightarrow 3a = -2d$

$a = -\frac{2}{3}d$

$q = \frac{-\frac{d}{3}}{\frac{d}{3}} = -1 \qquad 3a - 1 = -3$

$a, b, c \qquad 3b, \; 2a, \; c \qquad 4a = 3aq + aq^2 \Rightarrow$

$a \quad aq \quad aq^2 \qquad 3aq \quad 2a \quad aq^2 \qquad aq^2 + 3aq - 4a = 0$

$a(q^2+3q-4) = 0 \Rightarrow a \neq 0 \Rightarrow (q+4)(q-1) \qquad q \begin{cases} -4 \\ +1 \; \times \text{غیرقابل} \end{cases}$

$q^3 = (-4)^3 = -64$

$\frac{aq^5}{q^2q^2} + \frac{aq}{a^2} = 2 \Rightarrow \frac{q^2}{a^2} + \frac{q}{a} = 2 \Rightarrow \frac{q^2+aq}{a^2} = 2 \Rightarrow$

$2a^2 = q^2 + aq \qquad q^2 + aq - 2a^2 = 0 \qquad (q+2a)(q-a) \qquad q \begin{cases} -2a \\ +a \end{cases}$

$\frac{a^2}{aq} \begin{cases} 1 \\ -\frac{1}{2} \end{cases}$

$a_3 = aq^2 \qquad a_4 = a^2q^4 = aq^3 \Rightarrow aq = 1$

$aq^4 = 27 \Rightarrow \underbrace{aq}_{1} \times q^3 = 27 \Rightarrow q = 3 \qquad a \times 81 = 27 \qquad a = \frac{1}{3}$

$\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

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