پرنیان دهقانی

۱- اولین عدد ← ۸۱ دسته نهم $\{65, \dots, 81\}$ ۱۴۶ $1, \{2,3,4\}$

هشتمین ← ۶۴

$\frac{81+65}{2} = 73$ = واسطه حسابی

---

۲- $\{1,2,3\}, \{4,5,6,7,8,9,10,11,12\}$

$1\times 3 \quad 4\times 3 \quad 13\times 3 \quad 40\times 3 \quad 121\times 3$

عدد اول = ۱۲۱

عدد آخر = ۳۶۳

میانه $= \frac{121+363}{2} = \frac{484}{2} = 242$

---

۳- $2^k : n = 3k \to n = 3, 6, 9, 12, \dots \quad k = 1, 2, 3$

$-2k+4 : n = 3k+1 \to n = 1, 4, 7, 10, \dots \quad k = 0, 1, 2, 3, \dots$

$[\frac{n}{k+2}] + a : n = 3k+2 \to n = 2, 5, 8, 11, \dots \quad k = 0, 1, 2, 3, \dots$

$n=0 \to k=0 \to a_0 = 2^0 = 1$

$n=1 \to k=0 \to a_1 = 4 \;/\; n=2 \to k=0 \to a_2 = [\frac{2}{2}] + a = 1+a \;/\; n=3 \to a_3 = 2^1 = 2$

$n=4 \to k=1 \to a_4 = 2 \;/\; n=5 \to k=1 \to a_5 = [\frac{5}{3}] + a = 1+a \;/\; n=6 \to k=2 \to a_6 = 2^2 = 4$

$n=7 \to k=2 \to a_7 = 0 \;/\; n=8 \to k=2 \to a_8 = [\frac{8}{4}] + a = 2+a \;/\; n=9 \to k=3 \to a_9 = 2^3 = 8$

$a_0 + a_1 + \dots + a_9 = 1+4+1+a+2+2+1+a+4+0+2+a+8 = 25+3a = 19 \to 3a = -6 \to a = -2$

$a_2 + a_5 + a_8 + \dots + a_{29} = 1+a+1+a+8(2+a) = 18+10a = 18-20 = -2$

$[\frac{3k+2}{k+2}] = [\frac{k+2}{k+2} + \frac{2k}{k+2}] = [1 + \frac{2k}{k+2}] = 1 + [\frac{2k}{k+2}] \to [\frac{2k}{k+2}] \to \frac{2k}{k+2} \geq 0$

$\frac{3k+2}{k+2} = 1+0 = 1 \qquad n=1, n=5$

$\to 1+1 = 2 \qquad [n=8, n=11, n=14, \dots] \quad \frac{2k}{k+2} \geq 1 \to 2k \geq k+2 \qquad 2k \geq 0$

$k \geq 2 \qquad k \geq 0$

$\frac{2k}{k+2} \geq 2 \to 2k \geq 2k+4$

$0 \geq 4$ غ ق ق

ـ۴

$a_5 = 14$ $\quad a_7 = 17.2$ $\quad a = \frac{1}{70} \times (-14) = -\frac{1}{5} \to a_5 = -\frac{1}{5} = 25 + 5b + c = 14$

$a_n = -\frac{1}{5}n^2 + 4n - 1$

$-5 + 5b + c = 14$

$5b + c = 19$

$a_7 = -\frac{1}{5} \times 49 + 7b + c = 17.2 \to -9.8 + 7b + c = 17.2 \to 7b + c = 27$

$7b = c - 5b - c = 27 - 19 \to 2b = 8 \to b = 4 \to 5(4) + c = 19 \to c = -1$

$\frac{a_{15}}{a_1} = \frac{-\frac{1}{5} \times 15^2 + 15 \times 4 - 1}{-\frac{1}{5} \times 1^2 + 1 \times 4 - 1} = \frac{14}{\frac{14}{5}} = 5$

---

ـ۵

$a_4 = t_4$ $\quad a_8 = t_7$ $\quad t_{10} = 0$ $\quad a_4 + 4d = t_4 + 5d' \to 4d = 5d'$

$t_{15} = t_{10} + 5d' = 0 + 5d' = 5d' \to \frac{a_5}{d} = \frac{5d'}{d} = \frac{4d}{d} = 4$

---

ـ۶

$6a_2^2 = 5a_4 a + 3a_2 a \to 6a_2^2 - 3a_2 a = 5a_4 a \to 3a_2(2a_2 - a) = 5a_4 a \xrightarrow[a_2 = a_1 + d]{a_4 = a_1 + 3d}$

$3a_2(a_1 + 3d) = 5(a_1 + 3d)a_1$

$3a_2 = 5a_1 \to 3(a_1 + d) = 5a_1$

$3a_1 + 3d = 5a_1 \to 3d = 2a_1$

$a_1 + 3d = 0 \to a_1 = -3d$

$\frac{a_4}{d} = \frac{a_1 + 3d}{d}$

$a_1 = -3d$ : $\frac{3d - 3d}{d} = 1$

$3d = 2a_1$ : $\frac{a_1 + 3a_1}{\frac{2}{3}a} = \frac{9}{2}$

---

ـ۷

$c, b, a$ دنباله حسابی $\to b = c + d, a = c + 2d$

$\frac{c}{4}, \frac{a}{2}, b$ دنباله هندسی $\to \frac{a}{2} = \frac{c}{4} \times q, b = \frac{c}{4} \times q^2$

$c + d = \frac{cq^2}{4} \to \frac{cq^2}{4} - c = d$

$c(\frac{q^2}{4} - 1) = d$

$c + 2d = \frac{c}{4} \times q \times 2 \to c + 2d = \frac{cq}{2} \to \frac{cq}{2} - c = 2d \to c(\frac{q}{2} - 1) = 2d \to \frac{(\frac{q}{2} - 1)}{(\frac{q^2}{4} - 1)} = \frac{2d}{d}$

$\frac{1}{\frac{q}{2} + 1} = 2 \to 1 = q + 2 \to q = -1 \to 2a = -2$

---

۸

$b, a$ دنباله هندسی $\to b = cq, a = cq^2$

$2a, 3b$ دنباله حسابی $\to 2a = c + d, 3b = c + 2d$

$c + 2d = 3cq - c = 2d \to c(3q - 1) = 2d$

$2q^2 = c + d = 2cq^2 - c = d \to c(2q^2 - 1) = d$

$\frac{c(3q - 1)}{c(2q^2 - 1)} = \frac{2d}{d} = 2 \to 4q^2 - 2 = 3q - 1 \to 4q^2 - 3q - 1 = 0 \to \Delta = 9 + 16 = 25$

$q = \frac{3 \pm 5}{8} = 1$ یا $-\frac{1}{4} \to q = -\frac{1}{4}$

غ ق ق

$q^3 = -\frac{1}{64}$

$$\frac{a_4}{(a_2)^2} + \frac{a_3}{a^2} = 2 \to \frac{a_1 q^5}{a_1^2 q^2} - \frac{aq}{a_1^2} \to \frac{q^2}{a^2} + \frac{q}{a} = 2 \to \frac{q^2}{a_1^2} + \frac{q}{a_1} - 2 = 0$$ ۹

$$\frac{a_2}{a_4} = \frac{a_1}{a_1 q} = \frac{a_1}{q} \leftarrow \frac{q}{a} = 1 \quad \underline{\text{ل}}\ (-2) \qquad \left(\frac{q}{a}+2\right)\left(\frac{q}{a}-1\right) = 0$$

ب ۱ ل −۱۰

---

$$a_3^2 = a_4 \;,\; a_5 = 27 \qquad a_3^2 = a_2 q \to a_2 = q$$ ۱۰

$$a_5 = \frac{27}{a_2} = \frac{27}{q} \to q^2 \to 27 = q^3 \to q = 3 \qquad a_1 = \frac{a_5}{a_4} = \frac{27}{3^4} = \frac{27}{81} = \frac{1}{3}$$

$$\frac{1}{2} - a_1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$