

$$9^2 = 81 \Rightarrow \text{آخرین عدد در ۹}$$

$$48 \Rightarrow \text{اولین عدد} \Rightarrow 1^2 = 1 \Rightarrow \text{آخرین عدد در ۱}$$

$$\text{میانگین} = \frac{11+48}{2} = \frac{59}{2} = 29.5$$

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$$3, \dots, 12 \mid 13, \dots, 29 \mid 30, \dots, 48 \mid 49, \dots, 81$$

$$\text{میانگین} = \frac{343+121}{2} = 232$$

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$$a_0 + a_1 + a_2 + a_3 + \dots + a_9 = 19$$

$$a_0 = 1^2 = 1$$

$$a_1 + a_2 + a_3 + \dots + a_9 = 18$$

$$\left[\frac{1}{1}\right] + 1 + \left[\frac{2}{1}\right] + 1 + \left[\frac{3}{1}\right] + 1 + \dots$$

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$$\left[\frac{14}{1}\right] + 1 + \left[\frac{15}{1}\right] + 1 + \left[\frac{16}{1}\right] + 1 + \left[\frac{17}{1}\right] + 1 + \left[\frac{18}{1}\right] + 1 + \left[\frac{19}{1}\right] + 1 + \left[\frac{20}{1}\right] + 1 = 11$$

$$a_8 = 1^2$$

$$a_9 = 1^2$$

$$a_n = \frac{1}{x} \times (-a_0) x^r + b_n + C = -\frac{1}{x} x^r + b_n + C$$

$$\begin{aligned} \frac{1}{x} \times 1^2 + db + C &= 1^2 \\ \frac{1}{x} \times 1^2 + 1b + C &= 1^2 \end{aligned}$$

$$\frac{a_8}{a_1} = ? \rightarrow \frac{-\frac{1}{x} \times (1^2) + 1 \times x^r - 1}{-\frac{1}{x} + 1 - 1} = \frac{1^2}{1^2} = 1$$

$$\begin{aligned} db + C &= 1^2 \\ 1b + C &= 1^2 \\ \Rightarrow 1b &= 1 \quad b=1 \quad C=0 \end{aligned}$$

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$$t_k = a_1$$

$$d = \frac{t_1 - t_k}{r} = \frac{a_1 - a_r}{r}$$

$$t_1 = a_r$$

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$$a_{10} = 0$$

$$\frac{a_{10}}{d} = ? \rightarrow \frac{a_{10} + 0d}{a_{10} - 10d - a_{10} + 10d} = \frac{0}{0} = 1$$

$$\begin{aligned}
 ya^r &= \delta a + ra \quad y(a+rd)^r = \delta(a+rd)a + r(a+rd)a \\
 ra^r + ad &= yd^r \quad ra^r + ad - yd^r = 0 \\
 \frac{a^r}{d} &= \frac{a+rd}{d} = \frac{a}{d} + r = \frac{1}{\frac{d}{a}} + r = \frac{1}{x} + r \\
 \frac{1}{x} + r &= y \Rightarrow x^r + x - y = 0 \quad x = \frac{-1 \pm \sqrt{1+4y}}{2} \\
 x &= \frac{y}{y-1} \quad x = -1
 \end{aligned}$$

$$\begin{aligned}
 a, b, c &\rightarrow \text{geo} \quad rb = a + c \\
 b, \frac{a}{r}, \frac{c}{r} &\rightarrow \text{arith} \quad \frac{a^r}{r} = \frac{bc}{r} \Rightarrow a^r = bc \rightarrow b, a, c \rightarrow \text{arith} \\
 r_q &=? \\
 \frac{a}{a+d} &= \frac{a+rd}{a} \quad a^r = a^r + rad + rd^r \quad rad = -rd^r d \quad d = -\frac{ra}{r} \\
 b &= -\frac{a}{r} \quad c = -ra \quad q = \frac{\frac{a}{r}}{b} = \frac{a}{rb} = \frac{a}{-a} = -1 \quad r_q = -r
 \end{aligned}$$

$$\begin{aligned}
 a, b, c &\rightarrow \text{arith} \quad b^r = ac \\
 rb, ra, c &\rightarrow \text{geo} \quad ra = rb + c \\
 \frac{a_q}{a_r} &=? \quad \frac{aq^r}{aq^r} = q^r \quad ra = raq + aq^r \\
 q^r &= 1 \quad q^r = -4r \quad q^r + r_q - r = 0 \\
 q &= \frac{-r \pm \sqrt{r^2 + 4r}}{2} \Rightarrow q = 1 \quad q = -r
 \end{aligned}$$

$$\begin{aligned}
 \frac{a_q}{(a_r)^r} + \frac{a_r}{(a)^r} &= r \quad \frac{aq^r}{a^r q^r} + \frac{aq}{a^r a} = r \quad \frac{q^r + aq}{a^r} = r \\
 \frac{q^r}{a^r} + \frac{q}{a} - r &= 0 \quad \frac{q}{a} = x \Rightarrow x^r + x - r = 0 \\
 x &= \frac{-1 \pm \sqrt{1+4r}}{2} \quad x = -r \\
 x &= -r
 \end{aligned}$$

$$\begin{aligned}
 a_q &= \sqrt{a_r} = \sqrt{aq^r} = aq^r \quad aq^r = a^r q^r \quad aq = 1 \Rightarrow a_r = 1 \\
 a_q &= rV = aq^r = rV \\
 q^r &= \frac{rV}{1} \quad q = r \quad a = \frac{rV}{11} \\
 \frac{1}{r} - \frac{rV}{11} &= \frac{11 - r^2}{11r} = \frac{rV}{11r} = \frac{1}{4}
 \end{aligned}$$