مهسا نافراختی - دهم دختر ج - تکلیف شماره ی ۱۷!

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①

$\{1\},\{2,3,4\},\dots$

$\{1\},\{2,3,4\},\{5,6,7,8,9\},\dots,\{(n-1)^2+1,\dots,n^2\}$

$(n-1)^2+1 \qquad n^2 \qquad (n-1)^2+1 \qquad n^2$

$n=9 \to \{(n-1)^2+1,\dots,n^2\}=\{(9-1)^2+1,\dots,9^2\}$

$=\{65,\dots,81\} \to \dfrac{65+81}{2}=\dfrac{146}{2}=\boxed{73}$

②

$\{1,2,3\},\{4,5,6,7,8,9,10,11,12\},\{13,\dots,39\},$

$\{40,\dots,120\},\{121,\dots,363\},\dots$

$121+122+\dots+363$

تعداد $=\dfrac{363-121}{1}+1=242+1=243$

جمع $=\dfrac{(363+121)\times 243}{2}=\dfrac{484\times 243}{2}=242\times 243$

میانگین $=\dfrac{242\times 243}{243}=\boxed{242}$

③

$$a_n=\begin{cases} 2^k & ; n=3k \\ -2k+4 & ; n=3k+1 \\ \left[\dfrac{n}{k+2}\right]+a & ; n=3k+2 \end{cases}$$

$n=1 \to 1=3k+1 \to k=0 \to a_1=4$

$n=2 \to 2=3k+2 \to k=0 \to a_2=1+a$

$n=3 \to 3=3k \to k=1 \to a_3=2$

$n=4 \to 4=3k+1 \to k=1 \to a_4=2$

$n=5 \to 5=3k+2 \to k=1 \to a_5=1+a$

$n=6 \to 6=3k \to k=2 \to a_6=4$

$n=7 \to 7=3k+1 \to k=2 \to a_7=0$

$n=8 \to 8=3k+2 \to k=2 \to a_8=2+a$

$n=9 \to 9=3k \to k=3 \to a_9=8$

$n=0 \to 0=3k \to k=0 \to a_0=1$

③

$4+1+a+2+2+1+a+4+0+2+a+8+1=$

$25+3a=19 \to 3a=19-25 \to 3a=-6 \to a=-2$

$a_2=1+a=-1 \qquad -1-1=\boxed{-2}$

$a_5=1+a=-1$

$a_8=2+a=0$

$a_{11}=2+a=0$

$a_{14}=2+a=0$

$a_{17}=2+a=0$

$a_{20}=2+a=0 \quad \vdots$

④

$a_5=14 \qquad ax^2+bx+c$

$a_7=17.2 \qquad a=\dfrac{1}{70}(-14)=-\dfrac{14}{70}=-\dfrac{2}{10}=-\dfrac{1}{5}$

$-\dfrac{1}{5}x^2+bx+c$

$a_7=49a+7b+c=17.2 \qquad 12a+b=1.6$

$a_5=25a+5b+c=14 \qquad 12\left(-\dfrac{1}{5}\right)+b=\dfrac{16}{10}$

$49a+7b+c=17.2$

$-25a-5b-c=-14$

$24a+2b=3.2$

$-\dfrac{12}{5}+b=\dfrac{8}{5} \to b=\dfrac{20}{5}=4$

$-\dfrac{1}{5}x^2+4x+c$

$a_5=25\left(-\dfrac{1}{5}\right)+4\times 5+c=14$

$-5+20+c=14 \to 15+c=14 \to c=-1$

$-\dfrac{1}{5}x^2+4x-1$

$a_1=-\dfrac{1}{5}\times 1^2+4\times 1-1=-\dfrac{1}{5}+4-1=2\dfrac{4}{5}=\dfrac{14}{5}$

$a_{15}=-\dfrac{1}{5}(225)+4\times 15-1=-45+60-1=14$

$\dfrac{a_{15}}{a_1}=\dfrac{\frac{14}{1}}{\frac{14}{5}}$

$\dfrac{a_{15}}{a_1}=\boxed{5}$

⑤

$ax+b \rightarrow$ الگو خطی

$cn+d \rightarrow$ دنباله حسابی

$$4n+d = 2a+b \qquad 8n+d = 7a+b$$
$$8n+d = 7a+b \rightarrow \begin{array}{l} 8n+d = 7a+b \\ -4n-d = -2a-b \\ \hline 4n = 5a \end{array}$$

$$10a+b=0 \rightarrow b=-10a=-2(5a)=-2\times 4n=-8n$$
$$\rightarrow b=-8n$$

$$15a+b=3(5a)-8n=3\times 4n-8n=12n-8n=4n$$

$$8n+d-4n-d=4n \rightarrow \frac{4n}{4}=n$$ قدر نسبت دنباله حسابی

$$\frac{4n}{n}=\boxed{4}$$

⑥

$$6a_2^2 = 5a_3a + 3a_2a$$
$$6(a+d)^2 = 5(a+2d)a + 3(a+d)a$$
$$6(a^2+d^2+2ad) = 5a(a+2d) + 3a(a+d)$$
$$6a^2+6d^2+12ad = 5a^2+10ad+3a^2+3ad$$
$$6a^2+6d^2+12ad = 8a^2+13ad \rightarrow 6d^2 = 2a^2+ad$$
$$6d^2-ad-2a^2=0 \rightarrow d^2-ad-12a^2=0 \rightarrow (d-4a)(d+3a)=0$$
$$d-4a=0 \rightarrow d=\frac{4a}{6}=\frac{2a}{3} \rightarrow a=\frac{3}{2}d=1.5d$$
$$d+3a=0 \rightarrow d=-\frac{3a}{6}=-\frac{1}{2}a \rightarrow a=-2d$$
$$\frac{a_4}{d}=\frac{a+3d}{d}= \rightarrow \frac{-2d+3d}{d}=\frac{d}{d}=\boxed{1}$$
$$\hookrightarrow \frac{1.5d+3d}{d}=\frac{4.5d}{d}=\frac{45}{10}=\frac{9}{2}=\boxed{4.5}$$

⑦

$$a,\ b,\ c \qquad\qquad b,\ \frac{a}{2},\ \frac{c}{4}$$
$$a \downarrow (b-d) \qquad c \downarrow (b+d)$$

$$b,\ \frac{a}{2},\ \frac{c}{4} \rightarrow b,\ \frac{b-d}{2},\ \frac{b+d}{4}$$
$$\left(\frac{b+d}{4}\right)\times b = \left(\frac{b-d}{2}\right)^2 \rightarrow \frac{b^2+bd}{4}=\frac{b^2+d^2-2bd}{4}$$
$$\rightarrow b^2+bd=b^2+d^2-2bd \rightarrow 3bd=d^2 \rightarrow 3b=d$$
$$\frac{b+d}{4}=\frac{b+3b}{4}=\frac{4b}{4}=b$$
$$\frac{b-d}{2}=\frac{b-3b}{2}=\frac{-2b}{2}=-b$$
$$b \xrightarrow{\times(-1)} -b \xrightarrow{\times(-1)} b \qquad\qquad 2r=2\times(-1)=\boxed{-2}$$

⑧

$$a,\ b,\ c \qquad\qquad c,\ 2a,\ 3b$$
$$a \downarrow \frac{b}{r} \quad b \downarrow b \quad c \downarrow br \qquad c \downarrow br \quad 2a \downarrow \frac{2b}{r}$$

$$br+3b=2\times\frac{2b}{r} \rightarrow \cancel{b}(r+3)=\cancel{b}\left(\frac{4}{r}\right) \rightarrow r+3=\frac{4}{r}$$
$$r^2+3r=4 \rightarrow r^2+3r-4=0 \rightarrow (r+4)(r-1)=0$$
$r=-4$ ✓ $\qquad r=1$ ✗ غیرقابل قبول، جملات متمایز هستند.

$$\frac{a_9}{a_6}=\frac{a_6r^3}{a_6}=r^3=(-4)^3=\boxed{-64}$$

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$$\frac{a_6}{(a_2)^3}+\frac{a_2}{(a)^2}=2 \rightarrow \frac{ar^5}{(ar)^3}+\frac{ar}{a^2}=\frac{ar^5}{a^3r^3}+\frac{ar}{a^2}=2$$

$$\frac{r^2}{a^2}+\frac{r}{a}=2 \rightarrow \frac{r^2+ra}{a^2}=2 \rightarrow r^2+ra=2a^2 \rightarrow r^2+ra-2a^2=0$$

$$(r+2a)(r-a)=0 \rightarrow r=a \quad , r=-2a$$

$$\frac{a^2}{a_2}=\frac{a^2}{ar}=\frac{a}{r} \rightarrow \frac{a}{a}=\boxed{1}$$

$$\hookrightarrow \frac{a}{-2a}=\boxed{-\frac{1}{2}}$$

۱۰

$$a_3=\sqrt{a_4} \qquad a_3 , a_4 , a_5$$

$$a_5=27$$

$$a_3 \times a_5=a_4^2 \rightarrow \sqrt{a_4}\times 27=a_4^2 \rightarrow 27=\frac{a_4^2}{\sqrt{a_4}}\times\frac{\sqrt{a_4}}{\sqrt{a_4}} \rightarrow$$

$$27=\frac{a_4^2\sqrt{a_4}}{a_4}=a_4\sqrt{a_4}=(\sqrt{a_4})^3$$

$$27=(\sqrt{a_4})^3 \rightarrow \sqrt{a_4}=3 \rightarrow a_4=9$$

$$a_3 , a_4 , a_5 \rightarrow 3 \underbrace{,}_{\times 3} 9 \underbrace{,}_{\times 3} 27 \rightarrow \frac{1}{3}, 1, 3, 9, 27$$

$$a_1=\frac{1}{3} \rightarrow \frac{1}{2}-\frac{1}{3}=\frac{3-2}{6}=\boxed{\frac{1}{6}}$$