۱- $\frac{a}{q}, a, aq, \dots \to \frac{a}{q} \times a \times aq = 64 \to a^3 = 64 \to a = 4$

$\frac{a}{q} + a + aq = 21 \to \frac{4}{q} + 4 + 4q = 21 \to \frac{4}{q} + 4q - 17 = 0 \to 4q^2 - 17q + 4 = 0$

$q = \frac{-(-17) \pm \sqrt{(-17)^2 - 4(4\times4)}}{2(4)} \to q = \frac{17 \pm 15}{8} \to q = 4,\ \boxed{q = \frac{1}{4}}$

چون صعودی نیست پس $\boxed{q = \frac{1}{4}}$
این قابل قبول است

۲- $(2x)^2 = (x^2 - 2)(x^2 + 4) \to 4x^2 = x^4 + 2x^2 - 8 \to 2x^2 = x^4 - 8 \to x^4 - 2x - 8 = 0$

$(x^2 - 4)(x^2 + 2) = 0$

$x^2 = 4 \to x = 2 \to 8, 4, 2 \to q = \frac{1}{2}$

$x = \pm 2 \to x = -2 \to 8, -4, 2 \to$ غ‌ق‌ق $\to S_7 = 8 \times \frac{1 - (\frac{1}{2})^7}{1 - \frac{1}{2}} = \frac{127}{8}$

$x - \frac{1}{2}$

۳- $1 + q + q^2 + q^3 + q^4 = \frac{121}{81} \to S_5 = a_1 + a_1 q + a_1 q^2 + a_1 q^3 + a_1 q^4$

$= a_1(q + 1 + q^2 + q^3 + q^4) = 243 \times \frac{121}{81} = 363$

۴- حسابی: $A = \frac{1 + 64}{2} = 32/5$ ، هندسی: $B^2 = 1 \times 64 \to B = \pm 8$

(۱) $A + B = 32/5 + 8 = \boxed{40/5}$ ، (۲) $A + B = 32/5 - 8 = \boxed{24/5}$

۵- $a_{101} = a_1 + 100d \to d = \frac{-95}{4} - (-24) = \frac{1}{4} \to a_{101} = -24 + 100 \times \frac{1}{4} = 1$

$128, a_2, \dots \to a_n = 128 \times q^{n-1} \to a_8 = 128 \times q^7 = 1 \to q = \frac{1}{2}$

۶- $a_3 \quad a_7 \quad a_9$

$a_1 + 2d, a_1 + 6d, a_1 + 8d \to (a_1 + 6d)^2 = (a_1 + 2d)(a_1 + 8d) \to a_1^2 + 36d^2 + 12a_1 d$

$= a_1^2 + 16d^2 + 10a_1 d \to 20d^2 = -2a_1 d \to 2d(10d + a_1) = 0$

$2d = 0 \to \boxed{d = 0}$

$10d + a_1 = 0 \to d = \frac{-a_1}{10}$

/ /

a_2 , a_4 , a_8 7-

$a_1+d , a_1+3d , a_1+7d \to (a_1+3d)^2=(a_1+d)(a_1+7d) \to$

$a_1^2+9d^2+6a_1d = a_1^2+7d^2+8a_1d \to 2d^2=2a_1d \to d=a_1 \to 2a, 4a, 8a$

$q=\frac{4a}{2a}=2 \quad a_{10}=a_1q^9=\frac{1}{4}\times 2^9=128$

$3a_2 , 2a_3 , a_4$ 8-

$3(a_1q) , 2(a_1q^2) , a_1q^3 \quad 2a_1q^2=\frac{3a_1q+a_1q^3}{2} \to -4a_1q^2+3a_1q+a_1q^3=0$

$a_1q(-4q+3+q^2)=0 \to (q-1)(q-3)=0 \quad q=1$ غ ق ق

$a_1q=0 \quad \boxed{q=3}$

$a_1=0$ غ ق ق

$d=\frac{7}{4}-2=\frac{-1}{4} \quad a_4=2+3\times(-\frac{1}{4})=\frac{5}{4}$ 9-

$a_8=2+7\times(-\frac{1}{4})=\frac{1}{4}$

$a_{13}=2+12\times(-\frac{1}{4})=-1$

$\frac{5}{4}+x , \frac{1}{4}+x , -1+x \to (\frac{1}{4}+x)^2=(x+\frac{5}{4})(x-1) \to$

$\frac{1}{16}+x^2+\frac{1}{2}x=x^2+\frac{1}{4}x-\frac{5}{4} \to x=\frac{-21}{4} \to -4, -5, -\frac{25}{4}$

$q=\frac{-5}{-4}=\boxed{\frac{5}{4}}$

$a_1+a_1q^3+a_1q^6=73$ 10-

$t_1=a_1=a , t_2=a_4=aq^3 , t_{10}=a_7=aq^6$

$\left.\begin{array}{l} t_2=aq^3 \\ t_2=t_1+d \end{array}\right\} aq^3=a+d \to d=(q^3-1)a$

$t_{10}=aq^6=a+9a(q^3-1)$
$=a+9aq^3-9a \to aq^6-9aq^3+8a=0$
$a(q^6-9q^3+8) \to q^3=\frac{9\pm\sqrt{81-32}}{2}$

$q=1 \to a+a+a=73 \to a=\frac{73}{3}, d=a(q^3-1)$

$q^3=8 \to q=2$

$\frac{73}{3}\times(1-1)=\boxed{0}$

$q^3=1 \to q=1$

$q=2 \to a+8a+64a=73 \to a=1 \to d=a(q^3-1)=1\times(8-1)=\boxed{7}$