پاسخ سوال

(1)

$\frac{a}{q}\times a\times aq = 64 \to a^3 = 64$

$a = 4$

$\left(\frac{4}{q} + 4 + 4q = 21\right)\times q$

$\to 4 + 4q + 4q^2 \to 4q^2 - 17q + 4 = 0$

$q = \frac{17 \pm \sqrt{289-64}}{2\times 4}$ $<$ 4, $\frac{1}{4}$ $\qquad \boxed{q = \frac{1}{4}}$

(2)

$m^2+4,\ 2m,\ m^2-2 \to (2m)^2 = (m^2+4)(m^2-2)$

$m^4 - 2m^2 - 8 = 0$

$8,\ 4,\ 2 \to q = \frac{1}{2}$

$S_7 = a\left(\frac{q^n-1}{q-1}\right) = 8\left(\frac{(\frac{1}{2})^7-1}{\frac{1}{2}-1}\right) = 15{,}875$

$m^2 = \frac{2\pm\sqrt{4+32}}{2}$ $<$ 4, $\cancel{-2}$

(3)

$a_1 = 243$

$S_5 = \underbrace{243}_{a}\underbrace{(1+q+q^2+q^3+q^4)}_{\frac{121}{81}} = \boxed{363}$

۴)

واسطه حسابی: $\frac{55-1}{5+1} = 10{,}8$

$a_4 = a + (n-1)d = 1 + 3 \times 10{,}8 = 32{,}8$

واسطه هندسی: $q^{5+1} = 64 \rightarrow q = \pm 2$

$a_4 = aq^3 = 1 \times (\pm 2)^3 = 8$ or -8

$A + B = 40{,}8$ or $A + B = 24{,}8$

۵)

حسابی: $-24, -\frac{95}{4}, \dots$

هندسی: $128, 64, \dots$

a_{101} حسابی $= a_8$ هندسی

$a + (n-1)d = -24 + 100 \times \frac{1}{4} = 1$

$a_8 = 128 \times q^7 = 1 \rightsquigarrow q^7 = \frac{1}{128}$

$\boxed{q = \frac{1}{2}}$

۶)

$a_3 = a + 2d = a$ حسابی

$a_5 = a + 4d = aq$

$a_9 = a + 8d = aq^2$

$(a+4d)^2 = (a+8d)(a+2d) \Rightarrow a = -10d$

$\boxed{d = \frac{-a}{10}}$ or $\boxed{d = 0}$ $\rightsquigarrow$ چون جملات متمایز

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$a_2 = a + d = a$

$a_4 = a + 3d = aq$

$a_8 = a + 7d = aq^2$

$(a+3d)^2 = (a+d)(a+7d) \longrightarrow a = d$

$q = \frac{4a}{2a} = 2$

$\frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, 16, 32, 64, \underbrace{128}_{\text{جمله دهم}}$

8

$2aq, aq^2, aq^3$

$2aq^2 = \frac{2aq + aq^3}{2} \rightsquigarrow q(q^2 - 4q + 3) = 0$

$q = \frac{4 \pm \sqrt{16-12}}{2} \begin{cases} 3 \\ 1 \end{cases}$

$\boxed{q = 3}$

9

$\begin{cases} a_4 = 2 + 3 \times \frac{-1}{4} = 1.25 + d \\ a_8 = 2 + 7 \times \frac{-1}{4} = 0.25 + d \\ a_{13} = 2 + 12 \times \frac{-1}{4} = -1 + d \end{cases}$

$-4, -5, -6.25, \dots \quad \boxed{q = 1.25}$

$(0.25 + d)^2 = (1.25 + d)(d - 1) \rightsquigarrow \boxed{d = 5.25}$

$$a_1 + a_4 + a_7 = a + aq^3 + aq^6 = 73$$

$$a + a + 3d + a + 6d$$

$aq^3 = a + 3d$

$aq^6 = a + 6d$

$aq^6 = a(1 + 9(q^3 - 1)) = a(9q^3 - 8)$

$$q^6 = 9q^3 - 8 \begin{cases} q = 1 \\ q = 8 \end{cases}$$

$a(1 + 8 + 64) = 73$

$a = 1$ یا $3a = 73$

$a = \frac{73}{3}$

$d = aq^3 - a$

$\frac{73}{3} - \frac{73}{3} = 0$