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نام و نام خانوادگی .... [illegible] .... پاسخنامه تشریحی تکلیف شماره ..۱۵.. کلاس ......

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$\frac{x}{q} + x + xq = 21$, $\frac{x}{q} \times x \times xq = 64 \to x^3 = 4^3 \to x = 4$

$4,\ \frac{4}{q} + 4 + 4q = 21 \to \frac{4}{q} + 4q = 17 \to q = 4$ یا $q = \frac{1}{4} \to$ عددی غیرصحیح $q \to \boxed{q = \frac{1}{4}}$

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$x^2+4,\ 2x,\ x^2-2 \rightsquigarrow ac = b^2 \Rightarrow (x^2+4)(x^2-2) = (2x)^2$

$\rightsquigarrow x^4 - 2x^2 + 4x^2 - 8 = 4x^2 \Rightarrow x^4 - 2x^2 - 8 = 0 \to (x^2-4)(x^2+2) = 0 \begin{cases} x^2 = 4 \to x = \pm 2 \\ x^2 = -2\ ✗ \end{cases}$

$\rightsquigarrow x = +2 \rightsquigarrow 8, 4, 2 \to q = \frac{1}{2} \to S_n = \frac{a_1(1-q^n)}{(1-q)} \Rightarrow S_7 = \frac{8(1-(\frac{1}{2})^7)}{1-\frac{1}{2}}$

$\Rightarrow \frac{8(1-(\frac{1}{2})^7)}{\frac{1}{2}} = \boxed{\frac{127}{8}}$

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$S = a + aq + aq^2 + aq^3 + aq^4 \to a(1+q+q^2+q^3+q^4) \Rightarrow 243 \times \frac{121}{81} = \boxed{363}$

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$d = \frac{a_n - a_1}{\text{تعداد واسطه} + 1} = \frac{64-1}{6} = 10.5 \to A = a_4 = a_1 + 3d = 1 + 3 \times 10.5 = 32.5$

$aq^6 = 64 \to q^6 = 64 \to q = \pm 2 \to B = a_4 = a_1 \times q^3 \begin{cases} 1 \times (-2)^3 = -8 \\ 1 \times (+2)^3 = 8 \end{cases}$

$A + B = \begin{cases} 32.5 - 8 = 24.5 \\ 32.5 + 8 = 40.5 \end{cases}$

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$d = \frac{-95}{4} + 24 = \frac{1}{4} \to a_{101} = a_1 + (n-1)(\frac{1}{4}) = -24 + 100(\frac{1}{4}) \Rightarrow a_{101} = 1$

$b_n = a_1 q^{n-1} \to 1 = 128 \times q^7 \Rightarrow q^7 = \frac{1}{128} \to \boxed{q = \frac{1}{2}}$

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$a_3, a_7, a_9 \to a_1+2d, a_1+6d, a_1+8d \to ac=b^2$ $\Rightarrow (a_1+2d)(a_1+8d)=(a_1+6d)^2 \Rightarrow a_1^2+10a_1d+16d^2=a_1^2+36d^2+12a_1d$ $\Rightarrow -2a_1d+20d^2=0 \to 2d(a_1+10d)=0 \to a_1=-10d \to d=\frac{-a}{10}$, $d=0$ ۲	۶
$a_2, a_4, a_8 \to a_1+d, a_1+3d, a_1+7d \to ac=b^2$ $\Rightarrow (a_1+d)(a_1+7d)=(a_1+3d)^2 \Rightarrow a_1^2+8a_1d+7d^2=a_1^2+9d^2+6a_1d$ $\Rightarrow 2a_1d-2d^2=0 \to 2d(a_1-d)=0 \begin{cases} a_1=d \checkmark \\ d=0 \end{cases} \to a_2, a_4, a_8 = 2d, 4d, 8d$, $q \to \frac{4d}{2d}=2$ $a_{10}=a_1q^9 \Rightarrow \frac{1}{4}\times 2^9=\frac{512}{4}=128$ ۲	۷
$3a_2, 2a_3, a_4 \Rightarrow 3(a_1q), 2(a_1q^2), a_1q^3 \to a+c=2b$ $\Rightarrow 3a_1q+a_1q^3=2(2a_1q^2) \Rightarrow a_1(3q+q^3)=4a_1q^2 \Rightarrow 4q^2=3q+q^3 \to 4q=3+q^2$ $\Rightarrow q^2-4q+3=0$ دلتا $q=\frac{4\pm\sqrt{16-12}}{2} \Rightarrow \frac{4\pm\sqrt{4}}{2} \Rightarrow \begin{cases} (+) \to q=3 \\ (-) \to q=1 \end{cases}$ غیر ثابت $q=3$ ۲	۸
$2, \frac{7}{4}, \dots \to \frac{7}{4}-2=-\frac{1}{4}=d \to a_n=2-(n-1)\frac{1}{4} \Rightarrow a_4=\frac{5}{4}, a_8=\frac{1}{4}, a_{13}=-1$ $\frac{5}{4}+x, \frac{1}{4}+x, -1+x \Rightarrow b^2=ac \Rightarrow (-1+x)(\frac{5}{4}+x)=(\frac{1}{4}+x)^2$ $\Rightarrow -\frac{5}{4}-x+\frac{5}{4}x+x^2=\frac{1}{16}+x^2+\frac{1}{2}x \Rightarrow \frac{x}{4}=-\frac{21}{16} \Rightarrow x=-\frac{21}{4}$ $\Rightarrow -4, -5, -\frac{25}{4} \Rightarrow q=\frac{5}{4}$ ۲	۹
$a_1+a_4+a_7=73$, $a_1=b_1$, $a_4=b_2$, $a_7=b_{10}$ $a_4-a_1=b_2-b_1=d$, $a_7-a_1=b_{10}-b_1=9d \Rightarrow \frac{a(q^3-1)}{a(q^9-1)}=\frac{d}{9d} \Rightarrow q^3+1=9 \to q=2$ $a+aq^3+aq^6=a+8a+64a=73a=73 \Rightarrow a=1 \rightsquigarrow a_4-a_1=d \Rightarrow (1\times 2^3)-1=7 \to d=7$ $a_1+a_1q^3+a_1q^6=73 \Rightarrow 3a_1q^9=73 \Rightarrow q=3 \to 3a=73 \to a=\frac{73}{3}$ $aq^3-a=d \to \frac{73}{3}-\frac{73}{3}=0 \to d=0$ ۲	۱۰