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پیمانه‌ای (تکلیف ۵) رضوان پاکیزه c

(۱)

$$\frac{a}{q}+a+aq=\frac{a+aq+aq^{2}}{q}=21$$

$$\frac{a}{q}\times a\times aq=a^{3}=64 \rightarrow a=4$$

$$\Rightarrow \frac{4}{q}(1+q+q^{2})=21 \rightarrow 1+q+q^{2}=\frac{21}{4}q$$

$$\frac{4}{4}-\frac{21}{4}q=\frac{-17}{4}q$$

$$q^{2}-\frac{17}{4}q+1=0 \quad \times 4$$

$$4q^{2}-17q+4=0$$

$$\Delta=b^{2}-4ac$$

$$\Delta=289-4(4)(4)=225$$

$$x=\frac{-b\pm\sqrt{\Delta}}{2a}=\frac{17\pm15}{8}\rightarrow x=4$$

$q=x=\frac{1}{4}$ غیر قابل قبول

(۲)

$$x^{2}+4,\ 2x,\ x^{2}-2$$

$x=2 \rightarrow 8,4,2$ نزولی ($\times\frac{1}{2}=q$)

$x=-2 \rightarrow 8,-4,2$ ✗

$$4x^{2}=(x^{2}+4)(x^{2}-2)=x^{4}+2x^{2}-8$$

$$x^{4}-2x^{2}-8=0 \rightarrow (x^{2}-4)(x^{2}+2)=0$$

$x^{2}=4$ ، $x^{2}=-2$ ریشه حقیقی نداریم

$$x=\pm2$$

$$S_{7}=a_{1}\left(\frac{1-q^{7}}{1-q}\right)\rightarrow 8\left(\frac{1-(\frac{1}{2})^{7}}{1-\frac{1}{2}}\right)=16\left(\frac{128-1}{128}\right)=\frac{127}{8}$$

③

$1+q+q^2+q^3+q^4=\frac{121}{81}$

$a_1+a_2+a_3+a_4+a_5=a+aq+aq^2+aq^3+aq^4$

$=a(1+q+q^2+q^3+q^4)=243\left(\frac{121}{81}\right)=\boxed{363}$

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④

$d=\frac{64-1}{6}=\frac{63}{6}=10.5$

$q=-2$
$B=-8$
$A+B=24.5$
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$1, \bigcirc, \bigcirc, A, \bigcirc, \bigcirc, 64$ (×۲ ، ×۲ ، ×۲ → B ; +۱۰/۵)

$A=1+3(10.5)=32.5$
$B=1\times 2^3=8$
$\boxed{40.5}$

$q^6=\frac{64}{1}\Rightarrow q=2$

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⑤

$d=\frac{-95}{4}-(-24)=\frac{1}{4}$

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$a_{101}=-24+100\left(\frac{1}{4}\right)=\boxed{1}$

$a_8=128\times q^7=1\Rightarrow \boxed{q=\frac{1}{2}}$

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a_3 , a_7 , a_9

$a+2d , a+6d , a+8d$

$q = \frac{9-7}{7-3} = \frac{2}{4} = \frac{1}{2}$ ← تصحیح

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تصحیح

$(a+6d)^2 = (a+2d)(a+8d)$

$a^2 + 36d^2 + 12ad = a^2 + 10ad + 16d^2$

$20d^2 = -2ad$

$-10d = a \rightarrow d = \frac{-a}{10}$

[illegible]

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$a_{k_1} \quad a_{k_2} \quad a_{k_3}$

$a+d , a+3d , a+7d$

$2a \xrightarrow{\times 2} 4d \xrightarrow{\times 2} 8d \rightarrow q = 2$

$(a+3d)^2 = (a+d)\times(a+7d)$

$a^2 + 9d^2 + 6ad = a^2 + 8ad + 7d^2$

$2d^2 = 2ad$

$a = d$

$a_1 = a + d = 2a = \frac{1}{4} \rightarrow a = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

$a_{10} = \frac{1}{4}(2)^9 = \boxed{128}$

q^9

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$3a_2 , 2a_3 , a_4$

$3aq , 2aq^2 , aq^3$

$2aq^2 = \frac{3aq+aq^3}{2} \rightarrow -4aq^2 + 3aq + aq^3 = 0$

$\Rightarrow aq(-4q + 3 + q^2) = 0$

$aq = 0 \rightarrow a = 0$ غ ق ق

$\Rightarrow (q-1)(q-3) = 0 \nearrow q = 1$ غیرثابت ✗

$\searrow \boxed{q = 3}$

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$d = \frac{7}{4} - 2 = \frac{-1}{4}$

$a_4 = 2 + 3\left(-\frac{1}{4}\right) = \frac{5}{4}$

$a_8 = 2 + 7\left(-\frac{1}{4}\right) = \frac{1}{4}$

$a_{13} = 2 + 12\left(\frac{-1}{4}\right) = -1$

$\rightarrow \frac{5}{4} + x , \frac{1}{4} + x , -1 + x$

$\rightarrow \left(\frac{1}{4} + x\right)^2 = \left(x + \frac{5}{4}\right)(x - 1)$

$\Rightarrow$

$\frac{1}{16} + x^2 + \frac{1}{2}x = x^2 + \frac{1}{4}x - \frac{5}{4} \rightarrow x = \frac{-21}{4}$

$-4 , -5 , \frac{-25}{4} \rightarrow q = \frac{-5}{-4} = \boxed{\frac{5}{4}}$

$a_1 + aq^2 + aq^4 = 72$

$t_1 = a$

$t_2 = a + d = aq^3$

$t_{10} = a + 7d = aq^9$

$t_4 = aq^3$
$t_4 = 1 + d$
$\xrightarrow{a+}$ $aq^3 = a + d \rightarrow d = a(q^3 - 1)$

$t_{10} = aq^9 = a + 9a(q^3 - 1) = a + 9aq^3 - 9a$

$\Rightarrow aq^9 - 9aq^3 + 8a = 0 \rightarrow a(q^9 - 9q^3 + 8)$

$\Rightarrow q^3 = \frac{9 \pm \sqrt{81 - 32}}{2} \rightarrow \begin{cases} q^3 = 8 \Rightarrow q = 2 \\ q^3 = 1 \Rightarrow q = 1 \end{cases}$

$\rightarrow q = 1 \rightarrow \cancel{a + a + a} = 72 \rightarrow a = \frac{72}{3}$

$d = a(q^3 - 1) = \frac{72}{3}(1 - 1) = 0$

$q = 2 \rightarrow a + 8a + 64a = 72$

$a = 1 \rightarrow d = a(q^3 - 1)$

$d = 1(8 - 1) = 7$