نام و نام خانوادگی ......[illegible]...... پاسخنامه تشریحی تکلیف شماره ..۱۵.. کلاس ...۲...

۱

$\frac{a}{q} \times a \times aq = 64 \rightarrow a^3 = 64 \rightarrow a = 4$

$\frac{a}{q} + a + aq = 21 \rightarrow \frac{4}{q} + 4 + 4q = 21 \rightarrow q + \frac{1}{q} = \frac{17}{4} \rightarrow q^2 - \frac{17}{4}q + 1 = 0$

$\begin{cases} q = 4 \quad \times \\ \boxed{q = \frac{1}{4}} \end{cases}$

۲

$(x^2+4)(x^2-2) = 2x^2$

$x^4 - 2x^2 + 4x^2 - 8 = 4x^2$

$x^4 - 2x^2 - 8 = 0$

$\Delta = \frac{2 \pm \sqrt{4+32}}{2} = \frac{2 \pm 6}{2} \Rightarrow \Delta = 4 \rightarrow x = \pm 2 \rightarrow x = 2$

$\Delta = -2 \quad \times$

$a = t_1 = x^2 + 4 = 8$

$q = \frac{1}{2}$

$S_n = 8\left(\frac{(\frac{1}{2})^7 - 1}{\frac{1}{2} - 1}\right)$

$= 8\left(\frac{-\frac{127}{128}}{-\frac{1}{2}}\right)$

$= 8\left(\frac{254}{128}\right) = \boxed{15.875}$

۳

$S_5 = a_1(1 + q + q^2 + q^3 + q^4) \Rightarrow 243\left(\frac{121}{81}\right) = \boxed{363}$

۴

$1 \ \_ \ \_ \ \underline{A} \ \_ \ \_ \ 64$ ($+10.5$)

$d = \frac{64-1}{6} = 10.5$

$A = 1 + 3d$

$32.5 = 1 + 31.5$ ($\uparrow A$)

$1 \ \_ \ \_ \ \underline{B} \ \_ \ \_ \ 64$

$r^6 = 64$

$r = 2$

$B = 1 \times 2^3 = 2^3 = 8$

$A + B = 32.5 + 8 = 40.5$

۵

$-24, -\frac{95}{4}, \dots$

$d = -\frac{95}{4} - (-24) = \frac{1}{4}$

$t_{101} = -24 + 100\left(\frac{1}{4}\right)$

$= -24 + 25$

$= \boxed{1}$

$128, a_2, \dots$

$r^7 = \frac{128}{1}$

$\boxed{r = 2}$

۶

$t_3 = a+2d \quad t_7 = a+6d \quad t_9 = a+8d$

$t_7^2 = t_3 \times t_9$

$(a+6d)^2 = (a+2d)(a+8d)$

$2ad + 20d^2 = 0$

$2d(a+10d) = 0 \qquad a+10d=0 \qquad d = -\frac{a}{10}$

$q = \frac{t_7}{t_3} = \frac{a+6d}{a+2d} \xrightarrow{\text{با جایگذاری}} \frac{\frac{2a}{5}}{\frac{4a}{5}} = \frac{2}{4} = \frac{1}{2}$

۷

$t_2 = a+d$

$t_4 = a+3d$

$t_8 = a+7d$

$t_4^2 = t_2 \cdot t_8$

$(a+3d)^2 = (a+d)(a+7d)$

$a^2 + 6ad + 9d^2 = a^2 + 8ad + 7d^2 \longrightarrow 2d^2 - 2ad = 0$

$d = a$

$a+d = 2a \quad a+3d = 4a \quad a+7d = 8a$

$2a, 4a, 8a$ ($\times 2$, $\times 2$)

$q = \frac{4a}{2a} = 2$

$a_{10} = \frac{1}{4} \times 2^9 = \boxed{128}$

۸

$3ar, 2ar^2, ar^3$

$2ar^2 = \frac{3ar + ar^3}{2}$

$4ar^2 = 3ar + ar^3$

$4r^2 = 3r + r^3$

$r^3 - 4r^2 + 3r = 0 \longrightarrow r(r^2 - 4r + 3) = 0 \longrightarrow r(r-1)(r-3) = 0 \begin{cases} r=0 \ \times \\ r=1 \ \times \ \text{غیرثابت} \\ \boxed{r=3} \end{cases}$

۹

$2, \frac{7}{4}, \dots$

$d = \frac{7}{4} - 2 = -\frac{1}{4}$

$t_n = 2 - (n-1)\frac{1}{4}$

$t_4 = 2 - \frac{3}{4} = \frac{5}{4}$

$t_8 = 2 - \frac{7}{4} = \frac{1}{4}$

$t_{12} = 2 - \frac{12}{4} = -1$

$t_{12}+x, t_8+x, t_4+x$

$A = t_{12}+x = -1+x \quad B = t_8 + x = \frac{1}{4}+x \quad C = t_4 + x = \frac{5}{4} - x$

$B^2 = A \cdot C$

$(\frac{1}{4}+x) = (-1+x)(\frac{5}{4}+x) \longrightarrow x = -\frac{21}{4}$

$A = -1 - \frac{21}{4} = -\frac{25}{4} \quad B = \frac{1}{4} - \frac{21}{4} = -5 \quad C = \frac{5}{4} - \frac{21}{4} = -4$

$q = \frac{B}{A} = \frac{-5}{-\frac{25}{4}} = \boxed{\frac{4}{5}}$

۱۰

$t_1 = a \qquad A_1 = a$

$t_4 = ar^3 \qquad A_2 = ar^3$

$t_7 = ar^6 \qquad A_{10} = ar^6$

$a + ar^3 + ar^6 = 73$

$a(1 + r^3 + r^6) = 73$

$A_2 - A_1 = d$

$(ar^3 - a) = d$

$A_{10} - A_2 = 8d$

$(ar^6 - ar^3) = 8d \nearrow (ar^6 - ar^3) = 8(ar^3 - a)$

$r^6 - r^3 = 8r^3 - 8 \longrightarrow r^6 - 9r^3 + 8 = 0$

$x = \frac{9 \pm \sqrt{81-32}}{2} = \frac{9 \pm 7}{2} \begin{cases} x = 8 \\ x = 1 \ \times \end{cases}$

$x = r^3 \rightsquigarrow r^3 = 8 \quad r = 2$

$d = ar^3 - a = a(r^3 - 1) \downarrow a(8-1) = 7a$

$\frac{d}{A_1} = \frac{7a}{a} = \boxed{7}$