۲۵

(۴) $\frac{a}{q} + a + aq = 21$

$\frac{a}{q} \times a \times aq = 64 \rightarrow a = 4$

$\frac{4}{q} + 4 + 4q = 21 \rightarrow \frac{4}{q} + 4q = 17 \rightarrow 4q^2 + 4q - 17q = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{17 \pm \sqrt{289 - 64}}{8} = \begin{cases} 4 \\ \frac{1}{4} \end{cases}$

(۴) $(x^2 - 2)(x^2 + 6) = 4x^2$

$x^4 + 4x^2 - 8 = 4x^2 \rightarrow x^4 - 4x^2 - 8 = 0 \quad x^2 = t$

$\frac{4 \pm \sqrt{16 + 32}}{2} \rightarrow 4 = x^2 \rightarrow x = \pm 2$

$2 = x^2 \times -4$

$x = -2 \rightarrow 4, -8, 16$

$x = 2 \rightarrow 4, 8, 16 \rightarrow q = \frac{1}{2} \rightarrow a\left(\frac{q^n - 1}{q - 1}\right)$

$16 \times \frac{1 - \frac{1}{2^7}}{\frac{1}{2}} = 16 \times \frac{\frac{2^7 - 1}{2^7}}{\frac{1}{2}} = 16 \times \frac{(2^7 - 1) \times 2}{2^7 \times 2^6} = \frac{2^7 - 1}{2^6} = 2 - \frac{1}{64}$

۱۲۷۵

$\frac{127}{8}$

(۳) $a_1 + aq + aq^2 + aq^3 + aq^4 = 5$ ۳

$a(1 + q + q^2 + q^3 + q^4) = 3^3 \times \frac{121}{81} = 3 \times 121$

$\frac{121}{81}$

(۴) $\frac{a_{n-1}}{a_{n+1}} = \frac{45}{9} = 10/5 = d$ $A = a_5 = a_1 + 4d = 1 + 4 \times 10/5 = 43/5$

$\frac{45}{1} = 45 = q^4 \rightarrow q = \pm 3$ $B = a_4 = a_1(q^3) = 1 \times 8 = 8$

$\rightarrow 43/5 \pm 8 \rightarrow 43/5$, $23/5$ ۳ ↳ $1 \times -8 = -8$

(۵) $a_{101} = a_8^{15}$ ۳

$a + (n-1)d = -14 + 100 \times \frac{1}{2} = 1$ $a_8 = 128 \times q^7 = 1$

$q^7 = 128 \rightarrow q = \frac{1}{2}$

(۶) $a_3 = a + 2d = a \rightarrow (a+4d)^2 = a(a+8d)(a+2d)$

$a + 4d = aq$

$a + 8d = aq^2$

$a^2 + 16d^2 + 8ad = a^3 + 10da + 16d^2$

↳ $-2d^2 = -2da \rightarrow 10d = -a \rightarrow a = -10d$

$d = \frac{a}{-10}$ $d = 0$ ←

۳

(۷)

$a+d=a$

$a+3d=aq$

$a+7d=aq^2$

$\rightarrow (a+3d)^2=(a+d)(a+7d)$

$\rightarrow a^2+9d^2+6da=a^2+8da+7d^2$

$\rightarrow a=d$ — (۲)

$\dfrac{4d}{2d}=2=q \qquad a_1 q^9 = a_{10} = \dfrac{1}{4}\times 512 = 128$

---

(۸) — (۱۷۵)

$3aq,\ 2aq^2,\ aq^3$

$4aq^2=3aq+aq^3 \rightarrow q^3-4q^2+3q=0$

$q(q^2-4q+3)=0$

$q=\dfrac{4\pm\sqrt{16-12}}{2} \rightarrow 3$

$\rightarrow$ (1) → غیرقابل قبول چون دنباله ثابت می‌شود

---

(۹)

$a_4=2+3\times -\frac{1}{4}=1.25+d$

$a_8=2+7\times -\frac{1}{4}=\frac{1}{4}=0.25+d$

$a_{14}=2+13\times -\frac{1}{4}=-1+d$

$(0.25+d)^2=(-1+d)(1.25+d)$

$(0.25)^2+d^2+0.5d=-1.25-d+1.25d+d^2$

$\rightarrow 0.0625+1.25=-0.25d$

$\rightarrow d=-5.25$

$-4,\ -5,\ -6.25,\ \dots \rightarrow q=1.25$ — (۳)

---

(۱۰) — (۲)

$a_1+a_4+a_7=a+aq^3+aq^6=73$

(a, $a+3d$, $a+9d$)

$aq^3=a+d$

$aq^6=a+9d$ } $\rightarrow d=a(q^3-1)$

$aq^6=a(1+9(q^3-1))=a(9q^3-8)$

$q^6=9q^3-8 \rightarrow q=1$, $q^3=8$

$a(1+8+64)=73$

$73a=73 \rightarrow a=1$

$3a=73$

$\rightarrow a=\frac{73}{3}$

$d=aq^3-a=\frac{73}{3}-\frac{73}{3}=0$