آرین شیخان   دهم دفتران

۱- $\frac{a}{q}, a, aq, \dots \Rightarrow \frac{a}{q}\times a\times aq=64 \to a^3=64 \to a=4 \Rightarrow \frac{a}{q}+a+aq=21 \Rightarrow \frac{4}{q}+4+4q=21$

$\Rightarrow \frac{4}{q}+4q-17=0 \Rightarrow 4q^2-17q+4=0 \Rightarrow q=\frac{-(-17)\pm\sqrt{(-17)^2-4(4\times4)}}{2(4)} \Rightarrow q=\frac{17\pm15}{8}$

$\to$ 1) $q=4$
2) $q=\frac{1}{4}$ چون حسابجا نیست $\frac{1}{4}$ قابل قبول است

۲- $(2x)^2=(x^2-2)(x^2+4) \Rightarrow 4x^2=x^4+2x^2-8 \Rightarrow 2x^2-x^4-8 \Rightarrow (x^2-4)(x^2+2)=0$

$x^2=4 \Rightarrow 8,4,2 \Rightarrow q=\frac{1}{2}$

$S_7=8\times\frac{1-(\frac{1}{2})^7}{1-\frac{1}{2}}=8\times\frac{1-\frac{1}{128}}{\frac{1}{2}}=\frac{127}{8}$ $x=\pm2 \Rightarrow x=-2 \Rightarrow 8,-4,2=$ غ ق ق

۳- $1+q+q^2+q^3+q^4=\frac{121}{81}$, $S_5=a_1+a_1q+a_1q^2+a_1q^3+a_1q^4=a_1(1+q+q^2+q^3+q^4)$

$=243\times\frac{121}{81}=363$

۴- حسابی: $A=\frac{1+64}{2}=32.5 \to$ هندسی: $B^2=1\times64 \Rightarrow B=\pm8 \Rightarrow$
1) $A+B=32.5+8=40.5$
2) $A+B=32.5-8=24.5$

۵- $a_{101}=a_1+100d$, $d=\frac{-95}{4}-(-24)=\frac{1}{4} \Rightarrow a_{101}=-24+100\times\frac{1}{4}=1$

$128, a_2, \dots \Rightarrow a_n=128\times q^{n-1} \Rightarrow a_8=128\times q^{7}=1 \Rightarrow q=\frac{1}{2}$

۶-

a_3, a_7, a_9

$a_1+2d, a_1+6d, a_1+8d \Rightarrow (a_1+6d)^2=(a_1+2d)(a_1+8d) \Rightarrow a_1^2+36d^2+12a_1d=a_1^2+16d^2+10a_1d$

$\Rightarrow 20d^2=2a_1d \Rightarrow 2d(10d+a_1)=0$
1) $2d=0 \to d=0$
2) $10d+a_1=0 \to d=-\frac{a_1}{10}$

۷- a_2, a_4, a_8

$a_1+d, a_1+3d, a_1+7d \Rightarrow (a_1+3d)^2=(a_1+d)(a_1+7d) \Rightarrow a_1^2+9d^2+6a_1d=a_1^2+7d^2+8a_1d \Rightarrow$

$2d^2=2a_1d \Rightarrow d=a_1 \Rightarrow 2a, 4a, 8a \Rightarrow q=\frac{4a}{2a}=2 \to a_{10}=a_1q^9=\frac{1}{4}\times2^9=128$

8

$3a_2,\ 2a_3,\ a_4$

$3(a_1q),\ 2(a_1q^2),\ a_1q^3 \Rightarrow 2a_1q^2 = \dfrac{3a_1q + a_1q^3}{2} \Rightarrow -4a_1q^2 + 3a_1q + a_1q^3 = 0$

$\Rightarrow a_1q(-4q+3+q^2) = 0 \Rightarrow (q-1)(q-3) = 0 \Rightarrow (q-1)(q-3) = 0 \Rightarrow q=1$ غ ق ق

$q=3$

9

$d = \dfrac{7}{4} - 2 = -\dfrac{1}{4} \Rightarrow a_4 = 2 + 3\times\left(-\dfrac{1}{4}\right) = \dfrac{5}{4}$

$a_8 = 2 + 7\times\left(-\dfrac{1}{4}\right) = \dfrac{1}{4}$

$a_{13} = 2 + 12\times\left(-\dfrac{1}{4}\right) = -1$

$\rightarrow \dfrac{5}{4}+x,\ \dfrac{1}{4}+x,\ -1+x \Rightarrow \left(\dfrac{1}{4}+x\right)^2 =$

$\left(x+\dfrac{5}{4}\right)(x-1) \Rightarrow \dfrac{1}{16} + x^2 + \dfrac{1}{2}x = x^2 + \dfrac{1}{4}x - \dfrac{5}{4} \Rightarrow x = \dfrac{-21}{4} \Rightarrow -4,\ -5,\ \dfrac{-25}{4}$

$\Rightarrow q = \dfrac{\frac{-25}{4}}{\frac{-5}{1}} = \dfrac{5}{4}$

10

$a_1 + a_1q^3 + a_1q^6 = 73$

$t_1 = a_1 = a,\ t_2 = a_4 = aq^3,\ t_{10} = a_7 = aq^6$

$t_2 = aq^3$

$t_2 = \underset{a}{t_1} + d$

$aq^3 = a + d \rightarrow d = a(q^3-1)$

$t_{10} = aq^6 = a + 9a(q^3-1) = a + 9aq^3 - 9a \Rightarrow aq^6 - 9aq^3 + 8a = 0 \Rightarrow a(q^6 - 9q^3 + 8) \Rightarrow$

$q^3 = \dfrac{9 \pm \sqrt{81-32}}{2} \Rightarrow \begin{cases} q^3 = 8 \rightarrow q = 2 \\ q^3 = 1 \Rightarrow q = 1 \end{cases}$

$\rightarrow q=1 \rightarrow a+a+a = 73 \rightarrow a = \dfrac{73}{3}$

$q=2 \Rightarrow a + 8a + 64a = 73$

$a = \dfrac{73}{3} \Rightarrow d = a(q^3-1) = \dfrac{73}{3}\times(1-1) = 0$

$a = 1 \Rightarrow d = a(q^3-1) = 1\times(8-1) = 7$