نگار فتحی — به نام پروردگار نور — تکلیف شماره ۱۵ دهم (دختر) پنجشنبه

۱- $\frac{a}{q}, a, aq \qquad \frac{a}{q} \times a \times aq = a^3 = 64 \longrightarrow a = 4 \qquad \frac{4}{q} + 4 + 4q = 21$

$4 + 4q + 4q^2 = 21q \longrightarrow 4q^2 - 17q + 4 = 0 \longrightarrow q^2 - 17q + 16 = 0$

لم $(q-16)(q-1) \longrightarrow q = \cancel{\frac{16}{4}} , \frac{1}{4}$ غیرحسابی $q = \frac{1}{4}$

۲- $(x^2+4)(x^2-2) = 4x^2 \longrightarrow x^4 + 2x^2 - 8 = 4x^2 \longrightarrow x^4 - 2x^2 - 8 = 0$

$\frac{2 \pm \sqrt{4+32}}{2} = \frac{6}{2} \longrightarrow x^2 = -2 , 4 \quad \cancel{x^2 = -2} \quad x^2 = 4 \longrightarrow x = \pm 2 \xrightarrow{q>0} x = 2$

ترولی

$8, 4, 2, \ldots \longrightarrow S_7 = 8\left(\frac{1-\left(\frac{1}{2}\right)^7}{1-\frac{1}{2}}\right) = 16 \times \frac{127}{128} = \frac{127}{8}$

۳- $a(1+q+q^2+q^3+q^4) = a + aq + aq^2 + aq^3 + aq^4 = \frac{121}{81} \times a = 363$

۴- $d = \frac{64-1}{6} = \frac{63}{6}$ جمله چهارم (وسط) $= a + 3d \longrightarrow 1 + 3\left(\frac{63}{6}\right) = 32.5$

$\frac{64}{1} = q^6 \longrightarrow q = \pm 2 \quad \xrightarrow{q=2} aq^3 = 1 \times (2)^3 = 8 \quad \boxed{q=-2} \; aq^3 = 1 \times (-2)^3 = -8$

$A + B = 32.5 + 8 = 40.5$ یا $A + B = 32.5 - 8 = 24.5$

۵- $a_{101} = a + 100d \longrightarrow -24 + \left(100 \times \frac{1}{4}\right) = 1 \quad -\frac{95}{4} - (-24) = d = \frac{1}{4}$

$a_8 = aq^7 = 128 \times q^7 = 1 \longrightarrow q^7 = \frac{1}{128} \longrightarrow q = \frac{1}{2}$

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$a+2d, a+6d, a+8d \qquad (a+6d)^2=(a+2d)(a+8d)$

$a^2+36d^2+12ad=a^2+10ad+16d^2 \to 20d^2+2ad=0 \to 2d(10d+a)=0$

$d=0 \qquad 10d+a=0 \to 10d=-a \to d=-\frac{1}{10}a$

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$a+d, a+3d, a+7d \to (a+3d)^2=(a+d)(a+7d)$

$a^2+9d^2+6ad=a^2+8ad+7d^2 \to 2d^2-2ad=0 \to 2d(d-a)=0$

$\cancel{d=0} \quad d=a \qquad aq^9=\frac{1}{4}\times q^9 \qquad \underbrace{\underbrace{2d, 4d}_{\times 2}, 8d}_{q=2}$

مسایر $2^7=128$

$\frac{1}{4}\times(2)^9=128$

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$3aq, 2aq^2, aq^3 \to 4aq^2=3aq+aq^3$ نزولی

$q^2-4q+3=0 \to (q-1)(q-3) \to q=\cancel{1},3$

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$a+3d, a+7d, a+11d \qquad \frac{7}{4}-2=-\frac{1}{4} \to d$

$\frac{5}{4}$ ل $\quad \frac{1}{4}$ ل $\quad -1$ ل

$(\frac{1}{4}+x)^2=(\frac{5}{4}+x)(-1+x) \Rightarrow \frac{1}{16}+x^2+\frac{1}{2}x=-\frac{5}{4}+\frac{1}{4}x+x^2$

$\frac{1}{4}x=-\frac{21}{16} \to x=-\frac{21}{4} \qquad t_4=\frac{5}{4}-\frac{21}{4}=-4 \qquad t_8=\frac{1}{4}-\frac{21}{4}=-\frac{20}{4}=-5$

$q=\frac{-5}{-4}=\frac{5}{4}$

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$a, aq^3, aq^6 \to \underbrace{t, t+d}_{d}, \underbrace{t+9d}_{8d} \qquad \frac{1}{8}(aq^6-aq^3)=aq^3-a$

$8q^3-8=q^6-q^3$

$\to q^6-9q^3+8=0 \to (q^3-8)(q^3-1) \to q^3=8,1 \to q=2,1$

$a+aq^3+aq^6=73 \to a(1+q^3+q^6) \xrightarrow{q=1} a(1+1+1) \Rightarrow a=\frac{73}{3}$

$\xrightarrow{q=2} a(\cancel{1+2^3+2^6})=\cancel{73} \; a=1$

$q=1 \to t=73 \to \frac{73}{3}, \frac{73}{3}, \dots \to d=0$

$q=2 \to t=1 \to 1, 8, \dots \to d=7$