

$$a_1 + a_2 + a_3 = 21$$

$$a_1 \times a_2 \times a_3 = 4r \rightarrow \frac{a_2}{r} \times a_2 \times a_2 r = 4r$$

$$(a_2)^3 = (r)^3 \rightarrow a_2 = r$$

$$a_1 + a_2 + a_3 = \frac{r}{r} + r + r = 21 \rightarrow \frac{r}{r} + r = 14$$

$$\frac{r + r r^2}{r} = 14 \rightarrow r + r r^2 = 14r \rightarrow r r^2 - 14r + r = 0$$

$$r r^2 - 14r + r = 0 \rightarrow r^2 - 14r + 14 = 0 \rightarrow (r-1)(r-14) = 0$$

$$\boxed{q = \frac{1}{r}}$$

$$r = \frac{1}{r} \quad \downarrow \quad r = \frac{14}{r} = r \times$$

ر غیر حسابی باید باشد.

$$x^r + r, r x, x^r - r$$

$$(x^r + r)(x^r - r) = (r x)^r$$

$$x^r - r x^r + r x^r - r = r x^r \rightarrow x^r - r x^r - r = 0$$

$$x^r - r x^r - r = 0 \rightarrow (x^r - r)(x^r + r) = 0$$

$$\downarrow \quad \downarrow$$

$$x^r = r \quad x^r = -r x$$

$$\downarrow$$

$$x = r: \quad \frac{1}{r} \times \frac{1}{r} \quad x = \pm r$$

$$x = r: \quad 1, r, r \rightarrow q = \frac{1}{r}$$

$$x = -r: \quad 1, -r, r \quad \text{در صورتی که } x$$

$$S_v = a_1 \left(\frac{q^v - 1}{q - 1} \right) = 1 \left(\frac{\left(\frac{1}{r} \right)^v - 1}{\frac{1}{r} - 1} \right) = 1 \left(\frac{\frac{1}{r^v} - 1}{-\frac{1}{r}} \right) = 1 \left(\frac{1 - \frac{1}{r^v}}{-\frac{1}{r}} \right)$$

$$1 \left(\frac{1 - \frac{1}{r^v}}{-\frac{1}{r}} \right) = 1 \times \frac{1 - \frac{1}{r^v}}{-\frac{1}{r}} = \boxed{\frac{1 - \frac{1}{r^v}}{-\frac{1}{r}}}$$

$$1 + q + q^2 + q^3 + q^4 = \frac{121}{11}$$

$$1, q, q^2, q^3, q^4 \rightarrow S_5 = 1 \left(\frac{q^5 - 1}{q - 1} \right) = \left(\frac{q^5 - 1}{q - 1} \right)$$

$$\rightarrow \frac{q^5 - 1}{q - 1} = \frac{121}{11}$$

$$S_5 = a_1 \left(\frac{q^5 - 1}{q - 1} \right) = 1 \times \frac{121}{11} = 11$$

$$\boxed{S_5 = 11}$$

$$\text{حسابی: } 1, -, -, \frac{A}{r}, -, -, 4r$$

$$1 + 4d = 4r \rightarrow 4d = 4r - 1 \rightarrow 4d = 4r \rightarrow d = 1, 5$$

$$A = 1 + 3d = 1 + 3 \times 1, 5 = 1 + 3, 5 = 4, 5$$

$$\text{هندسی: } 1, -, -, \frac{B}{r}, -, -, 4r$$

$$1 \times r^4 = 4r \rightarrow r^4 = (\pm 1)^4 \rightarrow r = \pm 1$$

$$B = 1 \times r^3 \rightarrow 1 \times r^3 = 1$$

$$\hookrightarrow 1 \times (-1)^3 = -1$$

$$A + B = \rightarrow 4, 5 + 1 = \boxed{5, 5}$$

$$\hookrightarrow 4, 5 - 1 = \boxed{3, 5}$$

$$-2r, -\frac{9d}{r}, \dots \rightarrow -\frac{9d}{r}, -\frac{9d}{r}, \dots \rightarrow d = +\frac{1}{r}$$

$$a_{11} = a_1 + 10d = -2r + \frac{10}{r} = -2r + 10r = 8r = 1$$

$$a_n = a_1 \times r^{n-1} = 1 \rightarrow 1 \times r^{n-1} = 1 \rightarrow r^{n-1} = \frac{1}{1} = 1 \rightarrow r = 1$$

$$\boxed{r = 1}$$

$$a_r, a_v, a_q, \dots$$

$$a_r \times a_q = (a_v)^r$$

$$(a_v - f d) \times (a_v + r d) = a_v^r$$

$$a_v^r + r a_v d - f a_v d - \Lambda d^r = a_v^r$$

$$-r a_v d - \Lambda d^r = 0 \rightarrow -r a_v d = \Lambda d^r \rightarrow -r a_v = \Lambda d$$

$$-a_v = f d \rightarrow a_v = -f d$$

$$a_v = a_1 + 9d = -f d \rightarrow a_1 = -1 \cdot d$$

$$d = -\frac{a_1}{1} = -\frac{1}{1} a_1$$

$$d = 0$$

$$a_r, a_f, a_\Lambda, \dots$$

$$(a_f - r d)(a_f + f d) = a_f^r$$

$$a_f^r + r a_f d - \Lambda d^r = a_f^r \rightarrow r a_f d - \Lambda d^r = 0$$

$$r a_f d = \Lambda d^r \rightarrow r a_f = \Lambda d \rightarrow a_f = f d$$

$$r d, f d, \Lambda d, \dots \rightarrow r = f$$

$$a_{10} = a_1 \times r^9 = \frac{1}{f} \times r^9 = \frac{r^9}{r^r} = r^v = 12\Lambda \rightarrow a_{10} = 12\Lambda$$

$$r a_r, r a_r, a_r, \dots \rightarrow \frac{r a_r}{r}, r a_r, a_r$$

$$a_r r - r a_r = r a_r - \frac{r a_r}{r} \rightarrow a_r(r - r) = a_r(r - \frac{r}{r})$$

$$r - r = r - \frac{r}{r} \rightarrow \frac{r}{r} + \frac{r}{r} = r \rightarrow r^r + r - f r = 0 \rightarrow (r - r)(r - 1) = 0$$

$[r = r] \quad r = 1 \times \frac{r}{r}$

(4)

$$r, \frac{v}{f}, \dots \rightarrow \frac{\Lambda}{f}, \frac{v}{f}, \dots \rightarrow d = -\frac{1}{f}$$

$$a_f = a_1 + r d = r - \frac{r}{f} = 1 - \frac{1}{f} = \frac{f-1}{f}$$

$$a_\Lambda = a_1 + r d = r - \frac{v}{f} = \frac{\Lambda}{f} - \frac{v}{f} = \frac{\Lambda - v}{f}$$

$$a_{11} = a_1 + 11r d = r - \frac{11r}{f} = r - \frac{11}{f} = -1$$

$$a_{11} + x, a_\Lambda + x, a_f + x$$

$$-1 + x, \frac{1}{f} + x, \frac{f-1}{f} + x$$

$$\frac{f x - f}{f}, \frac{f x + 1}{f}, \frac{f x + f}{f}$$

$$\left(\frac{f x - f}{f}\right) \times \left(\frac{f x + f}{f}\right) = \left(\frac{f x + 1}{f}\right)^r$$

$$\frac{14 x^r + r \cdot x - 14 x - r}{14} = \frac{14 x^r + 1 + \Lambda x}{14}$$

$$f x - r = \Lambda x + 1 \rightarrow -r = \Lambda x + 1 \rightarrow x = -\frac{r+1}{\Lambda}$$

$$-1 + x = -1 - \frac{r+1}{f} = -\frac{f+r+1}{f} = -\frac{r+1}{f} \rightarrow (a_{11} + x)$$

$$\frac{1}{f} + x = \frac{1}{f} - \frac{r+1}{f} = -\frac{r}{f} \rightarrow (a_\Lambda + x)$$

$$r = \frac{a_{11} + x}{a_\Lambda + x} = \frac{-\frac{r+1}{f}}{-\frac{r}{f}} = \frac{r+1}{r} = \frac{f+1}{f}$$

(A)

$$a_1 + a_f + a_v = v^w$$

①

$$\frac{a_f}{r^w} + a_f + a_f r^w = v^w$$

$$\frac{a_v - a_f}{\Lambda} = a_f - a_1 \rightarrow \frac{a_f r^w - a_f}{\Lambda} = a_f - \frac{a_f}{r^w}$$

$$a_f \left(\frac{r^w}{\Lambda} - 1 \right) = a_f \left(1 - \frac{1}{r^w} \right)$$

$$\frac{r^w}{\Lambda} - 1 = \frac{r^w}{r^w} - 1 \rightarrow \frac{1}{\Lambda} = \frac{1}{r^w} \rightarrow \Lambda = r^w \rightarrow r = r$$

$$\frac{a_f}{\Lambda} + a_f + \Lambda a_f = v^w$$

$$a_f \left(\frac{1}{\Lambda} + 1 + \Lambda \right) = v^w \rightarrow a_f \left(\frac{v^w}{\Lambda} \right) = v^w \rightarrow a_f = \Lambda$$

$$d = a_f - a_1 = a_f - \frac{a_f}{r^w} = \Lambda - \frac{\Lambda}{r^w} = \Lambda - \frac{\Lambda}{\Lambda} = \Lambda - 1 = v$$

$$\boxed{d = v} \quad \boxed{d = 0}$$