ثمرانا فراغتی - دهم دختر C - تکلیف شماره ی ۱۵

۲۰

①

$a_1+a_2+a_3=21$

$a_1\times a_2\times a_3=64 \rightarrow \frac{a_2}{r}\times a_2\times a_2 r=64$

$(a_2)^3=(4)^3 \rightarrow a_2=4$

$a_1+a_2+a_3=\frac{4}{r}+4+4r=21 \rightarrow \frac{4}{r}+4r=17$

$\frac{4+4r^2}{r}=17 \rightarrow 4+4r^2=17r \rightarrow 4r^2-17r+4=0$

$4r^2-17r+4=0 \rightarrow r^2-17r+16=0 \rightarrow (r-1).(r-16)=0$

$\boxed{q=\frac{1}{4}}$ $\quad r=\frac{1}{4} \quad , \quad r=\frac{16}{4}=4 \times$

r غیرحسابی باید باشد.

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②

$x^2+4 \;,\; 2x \;,\; x^2-2$

$(x^2+4)\times(x^2-2)=(2x)^2$

$x^4-2x^2+4x^2-8=4x^2 \rightarrow x^4-2x^2-8=0$

$x^4-2x^2-8=0 \rightarrow (x^2-4)(x^2+2)=0$

$x^2=4 \quad\quad x^2=-2 \times$

$x=\pm 2$

$x=2: \; 8 \xrightarrow{\times\frac{1}{2}} 4 \xrightarrow{\times\frac{1}{2}} 2 \rightarrow q=\frac{1}{2}$

$x=-2: \; 8, -4, 2 \times$ نه نزولی است و نه صعودی

$S_7=a_1\left(\frac{q^7-1}{q-1}\right)=8\left(\frac{(\frac{1}{2})^7-1}{\frac{1}{2}-1}\right)=8\left(\frac{\frac{1}{128}-1}{-\frac{1}{2}}\right)=8\left(\frac{\frac{1-128}{128}}{-\frac{1}{2}}\right)$

$8\left(\frac{-\frac{127}{128}}{-\frac{1}{2}}\right)=8\times\frac{127\times 2}{128}=\boxed{\frac{127}{8}}$

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③

$1+q+q^2+q^3+q^4=\frac{121}{81}$

$1 \xrightarrow{\times q} q \xrightarrow{\times q} q^2 \xrightarrow{\times q} q^3 \xrightarrow{q} q^4 \rightarrow S_5=1\left(\frac{q^5-1}{q-1}\right)=\left(\frac{q^5-1}{q-1}\right)$

$\rightarrow \frac{q^5-1}{q-1}=\frac{121}{81}$

$S_5=a_1\left(\frac{q^5-1}{q-1}\right)=\cancel{243}^{3}\times\frac{121}{\cancel{81}}=3\times 121=363$

$\boxed{S_5=363}$

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④

حسابی: $1, \_, \_, A, \_, \_, 64$

$1+6d=64 \rightarrow 6d=64-1 \rightarrow 6d=63 \rightarrow d=10{,}5$

$A=1+3d=1+3\times 10{,}5=1+31{,}5=32{,}5$

هندسی: $1, \_, \_, B, \_, \_, 64$

$1\times r^6=64 \rightarrow r^6=(\pm 2)^6 \rightarrow r=\pm 2$

$B=1\times r^3 \rightarrow 1\times 2^3=8$

$\quad\quad \rightarrow 1\times(-2)^3=-8$

$A+B= \rightarrow 32{,}5+8=\boxed{40{,}5}$

$\quad\quad \rightarrow 32{,}5-8=\boxed{24{,}5}$

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⑤

$-24, -\frac{95}{4}, \ldots \rightarrow -\frac{94}{4}, -\frac{95}{4}, \ldots \rightarrow d=+\frac{1}{4}$

$a_{101}=a_1+100d=-24+\frac{100}{4}=-24+25=1$

$a_8=a_1\times r^7=1 \rightarrow 128\times r^7=1 \rightarrow r^7=\frac{1}{128}=\left(\frac{1}{2}\right)^7$

$\boxed{r=\frac{1}{2}}$

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$a_3 , a_7 , a_9 , \dots$

$a_3 \times a_9 = (a_7)^2$

$(a_7 - 4d) \times (a_7 + 2d) = a_7^2$

$\cancel{a_7^2} + 2a_7 d - 4a_7 d - 8d^2 = \cancel{a_7^2}$

$-2a_7 d - 8d^2 = 0 \rightarrow -2a_7 \cancel{d} = 8d^{\cancel{2}} \rightarrow -2a_7 = 8d$

$-a_7 = 4d \rightarrow a_7 = -4d$

$a_7 = a_1 + 6d = -4d \rightarrow a_1 = -10d$

$\boxed{d = -\frac{a_1}{10} = -\frac{1}{10} a_1}$

$\boxed{d = 0} \rightarrow$

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3

$a_2 , a_4 , a_8 , \dots$

$(a_4 - 2d)(a_4 + 4d) = a_4^2$

$\cancel{a_4^2} + 2a_4 d - 8d^2 = \cancel{a_4^2} \rightarrow 2a_4 d - 8d^2 = 0$

$2a_4 \cancel{d} = 8d^{\cancel{2}} \rightarrow 2a_4 = 8d \rightarrow a_4 = 4d$

$2d \xrightarrow{\times 2} 4d \xrightarrow{\times 2} 8d , \dots \rightarrow r = 2$

$a_{10} = a_1 \times r^9 = \frac{1}{4} \times 2^9 = \frac{2^9}{2^2} = 2^7 = 128 \rightarrow \boxed{a_{10} = 128}$

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2

$3a_2 , 2a_3 , a_4 , \dots \rightarrow \frac{3a_3}{r} , 2a_3 , a_3 r$

$a_3 r - 2a_3 = 2a_3 - \frac{3a_3}{r} \rightarrow \cancel{a_3}(r-2) = \cancel{a_3}(2 - \frac{3}{r})$

$r - 2 = 2 - \frac{3}{r} \rightarrow \frac{r^2}{r} + \frac{3}{r} = 4 \rightarrow r^2 + 3 - 4r = 0 \rightarrow (r-3)(r-1) = 0$

$\boxed{r = 3}$ $\quad r = 1 \times$ غیرثابت

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2

$2 , \frac{7}{4} , \dots \rightarrow \frac{8}{4} , \frac{7}{4} , \dots \rightarrow d = -\frac{1}{4}$

$a_4 = a_1 + 3d = 2 - \frac{3}{4} = 1\frac{1}{4} = \frac{5}{4}$

$a_8 = a_1 + 7d = 2 - \frac{7}{4} = \frac{8}{4} - \frac{7}{4} = \frac{1}{4}$

$a_{13} = a_1 + 12d = 2 - \frac{12}{4} = 2 - 3 = -1$

$a_{13} + x , a_8 + x , a_4 + x$

$-1 + x , \frac{1}{4} + x , \frac{5}{4} + x$

$\frac{4x-4}{4} , \frac{4x+1}{4} , \frac{4x+5}{4}$

$\left(\frac{4x-4}{4}\right) \times \left(\frac{4x+5}{4}\right) = \left(\frac{4x+1}{4}\right)^2$

$\frac{16x^2 + 20x - 16x - 20}{\cancel{16}} = \frac{\cancel{16x^2} + 1 + 8x}{\cancel{16}}$

$4x - 20 = 8x + 1 \rightarrow -21 = 4x \rightarrow x = -\frac{21}{4}$

$-1 + x = -1 - \frac{21}{4} = \frac{-4-21}{4} = -\frac{25}{4} \rightarrow (a_{13} + x)$

$\frac{1}{4} + x = \frac{1}{4} - \frac{21}{4} = -\frac{20}{4} \rightarrow (a_8 + x)$

$r = \frac{a_{13} + x}{a_8 + x} = \frac{-\frac{25}{\cancel{4}}}{-\frac{20}{\cancel{4}}} = \frac{25}{20} = \boxed{\frac{5}{4}}$

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$$a_{۱} + a_{۴} + a_{۷} = ۷۳$$

$$\frac{a_{۴}}{r^{۳}} + a_{۴} + a_{۴}r^{۳} = ۷۳$$

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$$\frac{a_{۷} - a_{۴}}{۸} = a_{۴} - a_{۱} \rightarrow \frac{a_{۴}r^{۳} - a_{۴}}{۸} = a_{۴} - \frac{a_{۴}}{r^{۳}}$$

$$\cancel{a_{۴}}\left(\frac{r^{۳}-۱}{۸}\right) = \cancel{a_{۴}}\left(۱ - \frac{۱}{r^{۳}}\right)$$

$$\frac{\cancel{r^{۳}-۱}}{۸} = \frac{\cancel{r^{۳}-۱}}{r^{۳}} \rightarrow \frac{۱}{۸} = \frac{۱}{r^{۳}} \rightarrow ۸ = r^{۳} \rightarrow r = ۲$$

$$\frac{a_{۴}}{۸} + a_{۴} + ۸a_{۴} = ۷۳$$

$$a_{۴}\left(\frac{۱}{۸} + ۱ + ۸\right) = ۷۳ \rightarrow a_{۴}\left(\frac{۷۳}{۸}\right) = ۷۳ \rightarrow a_{۴} = ۸$$

$$d = a_{۴} - a_{۱} = a_{۴} - \frac{a_{۴}}{r^{۳}} = ۸ - \frac{۸}{۲^{۳}} = ۸ - \frac{۸}{۸} = ۸ - ۱ = ۷$$

$\boxed{d = ۷}$ $\boxed{d = ۰}$