فاطمه حکیم الهی

بنام خدا

تکلیف ۱۴ - دهم ۵ شاهد دختران ۲

۱) الف) $a_{10}=2^{10-2}=2^8=\boxed{256}$

$q=2 \quad a_1=\frac{1}{2}$

$a_n=\frac{1}{2}(2)^{n-1}=2^{n-2}$

د) $a_n=2^{n-2}=128=2^7 \Rightarrow n-2=7 \Rightarrow n=9$

ب) $\frac{a_{17}}{a_{13}}=q^4=2^4=\boxed{16}$

ج) $\frac{a_{10}}{a_4}=q^6=2^6=64$

$64=q'^2 \Rightarrow q'=8 \Rightarrow$ واسطه $=a_4\times 8=4\times 8=\boxed{32}$

$a_4=\cancel{a_3}^{\,2}\times 2=4$

۲) $\frac{a_8}{a_5}=\frac{96}{12}=8=q^3 \Rightarrow q=2$

$a_5=12=a_1\times q^4=a_1\times 16=12 \Rightarrow a_1=\frac{3}{4}$

$a_{10}=a_1\times q^{10-1}=\frac{3}{\cancel{4}}\times \cancel{2^9}\,2^7=128\times 3=\boxed{384}$

۳) الف) $\underbrace{a_1\underbrace{a_2\,a_3\,a_4}_{a_2\times a_4=a_3\times a_3}a_5}_{a_1\times a_5=a_3\times a_3}=(a_3)^2\times(a_3)^2\times a_3=a_3^5=243 \Rightarrow a_3=\boxed{3}$

ب) $a_1\times a_5=(a_3)^2=3^2=9$

۴) $2^b\times 2^a=(4\sqrt{2})^2=16\times 2=2^5 \Rightarrow a+b=5$

$\frac{a+b}{2}=$ واسطه حسابی $=\frac{5}{2}=\boxed{2.5}$

۵)

$a_3=1-x=a_1q^2$

$a_7=x=a_1q^6$

$a_{11}=x+1=a_1q^{10}$

$a_3\times a_{11}=a_7\times a_7 \Rightarrow (1-x)(x+1)=x^2 \Rightarrow 1-x^2=x^2 \Rightarrow 2x^2=1 \Rightarrow x^2=\frac{1}{2}$

$\boxed{x=\pm\frac{\sqrt{2}}{2}}$

۶)

$a_1+a_4=a_1+a_1q^3=28$

$a_2+a_3=a_1q+a_1q^2=12$

$a_1+a_2+a_3+a_4=40$

$\frac{\cancel{a_1}(\cancel{1+q}^{\,1+q^2-q})}{\cancel{a_1}q(\cancel{1+q})}=\frac{28}{12}$

$12+12q^2-12q=28q$

$\Rightarrow 12q^2-40q+12=0$

$\Rightarrow 3q^2-10q+3=0$

روش ضرب $\Rightarrow q^2-10q+9=0$

$q=\frac{9}{3},\ \frac{1}{3}$

$S_4=a_1\left(\frac{q^4-1}{q-1}\right)$

$\Rightarrow a_1\left(\frac{3^4-1}{3-1}\right)=40$

$\Rightarrow a_1=1 \Rightarrow a_4=1\times q^3=1\times 3^3=\boxed{27}$ بیشترین

if $q=\frac{1}{3}$ $\quad S_4=a_1\left(\frac{(\frac{1}{3})^4-1}{\frac{1}{3}-1}\right)=40 \Rightarrow a_1\times\frac{40}{27}=40$

$\Rightarrow a_1=\boxed{27 \rightarrow \text{بیشترین}}$

$\Rightarrow a_4=27\times\left(\frac{1}{3}\right)^3=1$

۷)

$xq^{-1}+x+xq=13$

$a_1+a_2+a_3=13$

$\frac{x}{q}\times x\times xq=27 \Rightarrow x=3$

$\Rightarrow \frac{3}{q},\ 3,\ 3q \qquad \frac{3}{q}+3+3q=\frac{3q^2+3+3q}{q}=13$

$\Rightarrow 3q^2-10q+3=0 \overset{\text{روش ضرب}}{\Rightarrow} q^2-10q+9=0 \Rightarrow q=\frac{9}{3},\ \frac{1}{3}$

ادامه صفحه بعد

ادامه سوال ۷

if $q=\frac{1}{3} \Rightarrow \frac{x}{q}, x, xq \Rightarrow \frac{3}{\frac{1}{3}}, 3, 3\times\frac{1}{3} \Rightarrow \underset{a_1}{9}, \underset{a_2}{3}, \underset{a_3}{1}$

if $q=3 \Rightarrow \frac{x}{q}, x, xq \Rightarrow \frac{3}{3}, 3, 9 \Rightarrow \underset{a_1}{1}, \underset{a_2}{3}, \underset{a_3}{9}$

۸) الف)

$S_{10} = a_1\left(\frac{q^{10}-1}{q-1}\right) = \cancel{2}\times\left(\frac{3^{10}-1}{\frac{3-1}{\cancel{2}}}\right) = 3^{10}-1 = (3^5)^2-1 = (243)^2-1 = \boxed{59048}$

ب)

$P_{10} = a_1^{10}\times(q)^{\frac{n-1\times n}{2}} = a_1^{10}\times q^{\frac{9\times 10}{2}} = a_1^{10}\times q^{45} = \boxed{2^{10}\times 3^{45}}$

۹)

$\frac{x}{q^2} \quad \frac{x}{q} \quad x \quad xq \quad xq^2 \quad xq^3$

الف)

$(xq^2 - xq^3)(x - xq) = (xq - xq^2)^2 \Rightarrow$

$\cancel{x^2q^2} - \cancel{x^2q^3} - \cancel{x^2q^3} + \cancel{x^2q^4} = \cancel{x^2q^2} + \cancel{x^2q^4} - \cancel{2x^2q^3}$

$\Rightarrow$ صورت سوال اثبات شد

$xq - xq$

$xq - xq^2$

$xq^2 - xq^3$

$q' = \frac{xq - xq^2}{x - xq} = \frac{\cancel{x}q(\cancel{1-q})}{\cancel{x}(\cancel{1-q})} = \boxed{q}$

ب)

۱۰

الف) $S_n = a_1 + (a_1+d) + (a_1+2d) + \cdots + a_1 + (n-1)d$

$S_n = a_1 \times n + (d + 2d + \cdots + (n-1)d) \Rightarrow$

$d(1+2+3+\cdots+(n-1))$

$\downarrow$

$\frac{(n-1)(n)}{2}$

$S_n = a_1 n + d\,\frac{(n-1)(n)}{2} \longrightarrow S_n = \frac{n}{2}\left(2a_1 + d(n-1)\right)$

ب) $S_n = a_1 + \cancel{a_1 q} + \cancel{a_1 q^2} + \cdots + \cancel{a_1 q^{n-1}}$

$\times q$ $\quad qS_n = \cancel{a_1 q} + \cancel{a_1 q^2} + \cancel{a_1 q^3} + \cdots + a_1 q^n$

$S_n - qS_n = a_1 - a_1 q^n \Rightarrow S_n(1-q) = a_1(1-q^n) \Rightarrow$

$S_n = \frac{a_1(1-q^n)}{(1-q)}$

$\Rightarrow \boxed{S_n = \frac{a_1(q^n-1)}{(q-1)}}$