فاطمه حکیم الهی — به نام خدا — تکلیف ۱۴ — دهم دانشسرای دختران

۱۹/۷۵

① الف) $a_{10} = 2^{10-2} = 2^8 = \boxed{256}$ ۲

$q = 2 \quad a_1 = \frac{1}{2}$

$a_n = \frac{1}{2}(2)^{n-1} = 2^{n-2}$

د) $a_n = 2^{n-2} = 128 = 2^7 \Rightarrow n - 2 = 7 \Rightarrow n = 9$

ب) $\frac{a_{17}}{a_{13}} = q^4 = 2^4 = \boxed{16}$

ج) $\frac{a_{10}}{a_4} = q^6 = 2^6 = 64$

$64 = q^2 \Rightarrow q = 8 \Rightarrow$ واسطه $= a_4 \times 8 = 4 \times 8 = \boxed{32}$

$a_4 = a_3 \times 2 = 4$

② $\frac{a_8}{a_5} = \frac{96}{12} = 8 = q^3 \Rightarrow q = 2$ ۲

$a_5 = 12 = a_1 \times q^4 = a_1 \times 16 = 12 \Rightarrow a_1 = \frac{3}{4}$

$a_{10} = a_1 \times q^{10-1} = \frac{3}{4} \times 2^9 = 128 \times 3 = \boxed{384}$

③ الف) $a_1 a_2 a_3 a_4 a_5 = (a_3)^2 \times (a_3)^2 \times a_3 = a_3^5 = 243 \Rightarrow a_3 = \boxed{3}$ ۲

($a_2 \times a_4 = a_3 \times a_3$, $a_1 \times a_5 = a_3 \times a_3$)

ب) $a_1 \times a_5 = (a_3)^2 = 3^2 = 9$

④ $2^b \times 2^a = (4\sqrt{2})^2 = 16 \times 2 = 2^5 \Rightarrow a + b = 5$ ۲

$\frac{a+b}{2} =$ واسطه حسابی $= \frac{5}{2} = \boxed{2/5}$

⑤ $\boxed{x = \pm \frac{\sqrt{2}}{2}}$ ۱/۷۵

فقط $\frac{\sqrt{2}}{2}$ قابل قبوله!

$a_3 = 1 - x = a_1 q^2$

$a_7 = x = a_1 q^6$

$a_{11} = x + 1 = a_1 q^{10}$

$a_3 \times a_{11} = a_7 \times a_7 \Rightarrow (1-x)(x+1) = x^2 \Rightarrow 1 - x^2 = x^2 \Rightarrow 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2}$

⑥ ۲

$a_1 + a_4 = a_1 + a_1 q^3 = 28$

$a_2 + a_3 = a_1 q + a_1 q^2 = 12$

$a_1 + a_2 + a_3 + a_4 = 40$

$\frac{a_1(1+q^3)}{a_1 q(1+q)} = \frac{28}{12}$ ($1 + q^2 - q$)

$12 + 12q^2 - 12q = 28q$

$\Rightarrow 12q^2 - 40q + 12 = 0$

$\Rightarrow 3q^2 - 10q + 3 = 0$

روش ضرب $\Rightarrow q^2 - 10q + 9 = 0$

$q = \frac{9}{3}, \frac{1}{3}$

$S_4 = a_1(\frac{q^4 - 1}{q - 1})$

$\Rightarrow a_1(\frac{3^4 - 1}{3 - 1}) = 40 \Rightarrow a_1 = 1 \Rightarrow a_4 = 1 \times q^3 = 1 \times 3^3 = \boxed{27}$ بیشترین

if $q = \frac{1}{3}$: $S_4 = a_1(\frac{(\frac{1}{3})^4 - 1}{\frac{1}{3} - 1}) = 40 \Rightarrow a_1 \times \frac{40}{27} = 40$

$\Rightarrow a_1 = \boxed{27}$ بیشترین

$\Rightarrow a_4 = 27 \times (\frac{1}{3})^3 = 1$

⑦ ۲

$xq^{-1} + x + xq = 13$

$a_1 + a_2 + a_3 = 13$

$\frac{x}{q} \times x \times xq = 27 \Rightarrow x = 3$

$\Rightarrow \frac{3}{q}, 3, 3q \quad \frac{3}{q} + 3 + 3q = \frac{3q^2 + 3 + 3q}{q} = 13$

$\Rightarrow 3q^2 - 10q + 3 = 0 \Rightarrow$ (روش ضرب) $q^2 - 10q + 9 = 0 \Rightarrow q = \frac{9}{3}, \frac{1}{3}$

ادامه صفحه بعد

ادامه سوال ۷

if $q = \frac{1}{3} \Rightarrow \frac{x}{q}, x, xq \Rightarrow \frac{3}{\frac{1}{3}}, 3, 3 \times \frac{1}{3} \Rightarrow \underset{a_1}{9}, \underset{a_2}{3}, \underset{a_3}{1}$

if $q = 3 \Rightarrow \frac{x}{q}, x, xq \Rightarrow \frac{3}{3}, 3, 9 \Rightarrow \underset{a_1}{1}, \underset{a_2}{3}, \underset{a_3}{9}$

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۸) الف)

$$S_{10} = a_1\left(\frac{q^{10}-1}{q-1}\right) = 2 \times \left(\frac{3^{10}-1}{3-1}\right) = 3^{10}-1 = (3^5)^2-1 = (243)^2-1 = \boxed{59048}$$

ب)

$$P_{10} = a_1^{10} \times (q)^{\frac{(n-1)\times n}{2}} = a_1^{10} \times q^{\frac{9\times 10}{2}} = a_1^{10} \times q^{45} = \boxed{2^{10} \times 3^{45}}$$

$\frac{x}{q^2} \quad \frac{x}{q} \quad x \quad xq \quad xq^2 \quad xq^3$

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۹) الف)

$x - xq$

$xq - xq^2$

$xq^2 - xq^3$

$$(xq^2 - xq^3)(x - xq) = (xq - xq^2)^2 \Rightarrow$$

$$x^2q^2 - x^2q^3 - x^2q^3 + x^2q^4 = x^2q^2 + x^2q^4 - 2x^2q^3$$

$\Rightarrow$ صورت سوال اثبات شد

ب)

$$q' = \frac{xq - xq^2}{x - xq} = \frac{xq(-q+1)}{x(1-q)} = \boxed{q}$$

(و) (١٠)

$S_n = a_1 + (a_1+d) + (a_1+2d) + \cdots + a_1 + (n-1)d$ (الف

$S_n = a_1 \times n + (d + 2d + \cdots + (n-1)d)$ $\Rightarrow$

$d(1+2+2^2+\cdots+(n-1))$

$\frac{(n-1)(n)}{2}$

$S_n = a_1 n + d\frac{(n-1)(n)}{2} \longrightarrow S_n = \frac{n}{2}(2a_1 + d(n-1))$

$S_n = a_1 + a_1q + a_1q^2 + \cdots + a_1q^{n-1}$ (ب

$\times q$

$qS_n = a_1q + a_1q^2 + a_1q^3 + \cdots + a_1q^n$

$S_n - qS_n = a_1 - a_1q^n \Rightarrow S_n(1-q) = a_1(1-q^n) \Rightarrow$

$S_n = \frac{a_1(1-q^n)}{(1-q)}$

$\Rightarrow S_n = \frac{a_1(q^n-1)}{(q-1)}$