

نام و نام خانوادگی: پاسخنامه تشریحی تکلیف شماره ۱۴ ... کلاس: ...

$\frac{1}{r}, 1, r, \dots, a_n = \frac{1}{r} \times (r)^{n-1}$ الف) $a_{10} \rightarrow \frac{1}{r} \times r^9 = r^{-1} \times r^9 = r^8$ ب) $\frac{a_{14}}{a_{12}} = \frac{\frac{1}{r} \times r^{14}}{\frac{1}{r} \times r^{12}} = r^2 = 14$ ج) $a_4 = \frac{1}{r} \times r^3 = r^2$ و به واسطه میانه: $r^2, r^1, r^0 \rightarrow r^2 = r^0 \rightarrow r = 1$ و به واسطه میانه: $a_{10} = \frac{1}{r} \times r^9 = r^8$ $d) r^{-1} \times r^{n-1} = r^{n-2} = 128 = r^7 \rightarrow n-2=7 \rightarrow \boxed{n=9}$	۱
$a_5 = a_1 r^4 = 12 \rightarrow \frac{a_8}{a_5} = \frac{a_1 r^7}{a_1 r^4} = \frac{94}{12} \rightarrow r^3 = 8 \rightarrow r = 2, a_1 = \frac{12}{14} = \frac{3}{7}$ $a_{10} = a_1 r^9 \rightarrow \frac{3}{7} \times 2^9 = 3 \times 2^8 = 3 \times 128 = \boxed{384}$	۲
a_1, a_2, a_3, a_4, a_5 $\frac{a_1}{r}, \frac{a_2}{r}, \frac{a_3}{r}, \frac{a_4}{r}, \frac{a_5}{r} \Rightarrow \frac{a_1}{r} \times \frac{a_2}{r} \times \frac{a_3}{r} \times \frac{a_4}{r} \times \frac{a_5}{r} = \frac{a_1^5}{r^5} = 243 \rightarrow \sqrt[5]{243} = 3 \Rightarrow a_1 = r = 3$ ب) $a_1 \times a_5 = \frac{a_1}{r} \times \frac{a_5}{r} = \frac{a_1^2}{r^2} = 9$	۳
$r^a, \sqrt[r]{r}, r^b$ $(\sqrt[r]{r})^r = r^a \times r^b \rightarrow (r^{\frac{1}{r}})^r = r^a \times r^b \rightarrow r^1 = r^{a+b} \rightarrow a+b=1 \xrightarrow{\text{میانگین هندسی}} \frac{a+b}{2} = \frac{1}{2}$	۴
t_u, t_v, t_{11} $u+1, u, 1-u \xrightarrow{\text{میانگین هندسی}} u^2 = (1-u)(u+1) \rightarrow u^2 = u+1-u^2-u \rightarrow u^2 = 1-u^2$ $u = \pm \frac{1}{\sqrt{2}} \leftarrow u = \pm \frac{\sqrt{2}}{2} \leftarrow u^2 = \frac{1}{2} \leftarrow 2u^2 = 1$	۵

$a_1 + a_1 r^3 = 18 \Rightarrow a_1 (1 + r^3) = 18 \rightarrow a_1 = \frac{18}{1+r^3}$ $a_1 r + a_1 r^5 = 12 \Rightarrow a_1 (r + r^5) = 12 \rightarrow a_1 = \frac{12}{r+r^5}$ $\frac{18}{1+r^3} = \frac{12}{r+r^5} \rightarrow 18(r+r^5) = 12(1+r^3) \Rightarrow 3r + 3r^5 = 2 + 2r^3$ $r = 3 \Rightarrow a_1 = \frac{12}{3+3^5} = \frac{12}{244} = \frac{3}{61}$ $r = \frac{1}{3} \Rightarrow a_1 = \frac{12}{\frac{1}{3} + \frac{1}{3^5}} = \frac{12}{\frac{1+1}{3^5}} = \frac{12}{\frac{2}{243}} = 1458$	6
$\frac{a}{r}, a, ar \rightarrow ar^3 = 27 \rightarrow a = 3 \rightarrow \frac{a}{r}, a, ar$ $\frac{a}{r} + a + ar = 13 \rightarrow \frac{a + ar + ar^3}{r} = 13 \rightarrow a + ar + ar^3 = 13r$ $r = \frac{1}{3} \rightarrow [9, 3, 1]$ $r = 3 \rightarrow [1, 3, 9]$	7
$S_n = a_1 \left(\frac{a^n - 1}{a - 1} \right) \rightarrow 1 \times \left(\frac{10 - 1}{1 - 1} \right) = 10 - 1$ $\rightarrow (1 \times 10^9) \frac{1}{1} \rightarrow (1 \times 10^9) \frac{1}{1} = (1 \times 10^9) \frac{1}{1} = 10^9 \times 1 = 10^9$ $a_{10} = 1 \times 10^9$	8
$a_1, a_1 q, a_1 q^2, a_1 q^3, \dots$ $a_1 q - a_1, a_1 q^2 - a_1 q, a_1 q^3 - a_1 q^2, \dots$ $a_1(q-1), a_1 q(q-1), a_1 q^2(q-1), \dots \Rightarrow$ $\frac{a_1(q-1)}{q} = q$	9
$S_n = \frac{n}{p} (pa_1 + (n-1)d)$ $S_n = a_1 + ar + ar^2 + \dots + a_{n-1} + a_n$ $\oplus S_n = a_n + a_{n-1} + a_{n-2} + \dots + ar + a_1$ $pS_n = (a_1 + a_n)(ar + a_{n-1}) + \dots + (a_n + a_1)$ $pS_n = n(a_1 + a_n) \rightarrow S_n = \frac{n}{p} (a_1 + a_n)$	$S_n = a \left(\frac{q^n - 1}{q - 1} \right)$ $S_n = a_1 + a_1 r + a_1 r^2 + \dots + a_1 r^{n-1}$ $\times r \ominus S_n r = a_1 r + a_1 r^2 + a_1 r^3 + \dots + a_1 r^n$ $S_n - S_n r = a - a_1 r^n$ $S_n (1 - r) = a(1 - r^n) \rightarrow S_n = a \left(\frac{1 - r^n}{1 - r} \right)$ $\Rightarrow S_n = a \frac{1 - r^n}{1 - r} \Rightarrow S_n = a \left(\frac{r^n - 1}{r - 1} \right)$