باران دیزجی سوم دفتر ۲ تکلیف ۱۴

۱)

الف) $a_n: \frac{1}{2}, 1, 2$

$q=2 \qquad a_n=\frac{1}{2}\times(2)^{n-1}$

$a_{10}=\frac{1}{2}(2)^{9} \Rightarrow$

$2^8=256$

ب) $\frac{a_{15}}{a_{11}} = q^4$

$2^4 = 16$

ج) a, b, c

$b^2=a.c$

$a_4=\frac{1}{2}(2)^3=4$

$a_{10}=256$

$b^2=1024$

$b=\pm 32$

د) $\frac{1}{2}(2)^{n-1}=128$

$2^{n-2}=128$

$n-2=7$

$n=9$

۲)

$aq^4=12 \qquad aq^7=96 \qquad q^3=\frac{96}{12} \qquad q^3=8 \qquad q=2$

$a_{10}=a_5\times q^5 \Rightarrow 12\times 32=384$

۳)

الف) $(a_3)^5=243 \qquad a_3=3$

$a_1.a_2.a_3.a_4.a_5=243$

ب) $(a_1.a_5)=(a_3)^2 \Rightarrow$

$(a_1.a_5)=9$

۴)

$2^a, 4\sqrt{2}, 2^b$

$a \quad b \quad c$

$b^2=a.c \Rightarrow 32=2^{b+a} \Rightarrow 2^5=2^{b+a}$

$b+a=5$ واسطه‌ی عددی، $2c=a+b \Rightarrow 2c=5$

$c=2/5$

۵)

$\frac{3}{?}, \frac{7}{?}, 11 \Rightarrow$ حسابی

$1-x, x, x+1 \Rightarrow x^2=(1-x)(x+1) \Rightarrow x^2=\cancel{x}+1-x^2-\cancel{x}$

$2x^2=1 \qquad x^2=\frac{1}{2}$

$x=\pm\frac{1}{\sqrt{2}}$

$a + aq^3 = 28 \Rightarrow a(1+q^3) = 28$

$aq + aq^2 = 12 \Rightarrow aq(1+q) = 12 \Rightarrow \dfrac{a(1+q^3)}{aq(1+q)} = \dfrac{7}{3} \Rightarrow$

$\dfrac{(1+q)(1+q^2-q)}{q(1+q)} = \dfrac{7}{3} \Rightarrow 3 + 3q^2 - 3q = 7q \Rightarrow 3q^2 - 10q + 3 = 0$

$q^2 - 10q + 9 = 0 \Rightarrow (q-1)(q-9) \Rightarrow \begin{cases} q = \frac{1}{3} \rightarrow \\ q = 3 \rightarrow \end{cases} \quad a(1+27) = 28 \quad a = 1$

[illegible] $\boxed{aq^3 = 27}$

$\dfrac{a}{q} \times a \times aq = 27 \quad a^3 = 27 \quad a = 3 \quad \left(\dfrac{3}{q} + 3 + 3q = 13\right) \times q \Rightarrow$

$3 + 3q + 3q^2 = 13q \Rightarrow 3q^2 - 10q + 3 = 0 \quad q^2 - 10q + 9 = 0 \quad (q-1)(q-9)$

$q \begin{cases} \frac{1}{3} \rightarrow & 1, 3, 9 \\ 3 \rightarrow & 9, 3, 1 \end{cases}$

الف)

$a_n = 2, 6, 18, \ldots$

$q = 3 \quad a_1 = 2 \quad S_n = \dfrac{a(q^n - 1)}{q-1} \Rightarrow$

$S_{10} = \dfrac{\cancel{2}(3^{10} - 1)}{\cancel{2}} = \boxed{3^{10} - 1}$

ب)

$a_{10} = 2 \times 3^9 \quad a_1 = 2$

$P_{10} = (4 \times 3^9)^5 \Rightarrow 2^{10} \times 3^{45}$

ب) $a, aq, aq^2, aq^3, \ldots, aq^{n-1}$

جدید: $aq - a, \; aq^2 - aq, \; aq^3 - aq^2, \; \ldots, \; aq^{n-1} - aq^{n-2}$

$\underbrace{a(q-1), aq(q-1)}_{\times q}, \underbrace{aq^2(q-1)}_{\times q}, \ldots$ تشکیل یک دنباله هندسی می‌دهد

با جمله اول $a(q-1)$ و قدر نسبت q

$\boxed{aq^{n-2}(q-1)} = aq^{n-1}$

$a_1 = a(q-1) \quad q = q$

الف ت)

$a_1, a_2, a_3, a_4, \ldots, a_n$

$a + (a+d) + (a+2d) + \ldots + (a+(n-1)d)$

$\dfrac{a + a + (n-1)d}{2} \times n \Rightarrow$

$(2a + (n-1)d) \times \dfrac{n}{2}$

ب ت)

$S = a_1 + a_2 + a_3 + \ldots + a_n$

$(S = a + \cancel{aq} + \cancel{aq^2} + \ldots + aq^{n-1}) \times q$

$qS = \cancel{aq} + \cancel{aq^2} + \cancel{aq^3} + \ldots + aq^n$

$S - Sq = a - aq^n \Rightarrow$

$S(1-q) = a(1-q^n) \Rightarrow S = \dfrac{a(1-q^n)}{1-q}$

$S = \dfrac{a(q^n - 1)}{q-1}$