

نام و نام خانوادگی پاسخنامه تشریحی تکلیف شماره ۱۱۴ کلاس
 ۲۵

$$a_n = \frac{1}{r}, 1, 2, \dots \rightarrow r=2 \quad a_n = \frac{1}{r} \times (r)^{n-1}$$

ا) $a_{10} = \frac{1}{r} \times r^9 = r^9 \times r^{-1} = r^8 = 256$

ب) $\frac{a_{10}}{a_1} = \frac{\frac{1}{r} \times r^{10}}{\frac{1}{r} \times r^1} = r^9 \times r^{-1} = r^8 = 256$

ج) $a_4 = \frac{1}{r} \times r^3 = r^3 \times r^{-1} = r^2 = 4 \Rightarrow b^2 = ac \rightarrow b^2 = r^2 \times r^2 = r^4 \rightarrow b^2 = (r^2)^2 \rightarrow b = r^2 = 4$

د) $\frac{1}{r} \times r^{n-1} = \frac{1}{r} \times r^9 \rightarrow r^{n-1} = r^9 \rightarrow n-1=9 \rightarrow n=10$

$$a_5 = 12 \quad a_1 = 96 \quad \frac{a_5}{a_1} = \frac{12}{96} \rightarrow r^4 = \frac{1}{8} \rightarrow r = \frac{1}{2}$$

$$a_{10} = 12 \quad a_1 = 96 \quad \frac{a_{10}}{a_1} = \frac{12}{96} = \frac{1}{8} = r^9 \rightarrow r = \frac{1}{2}$$

$$a_{10} = a_1 \times r^9 = \frac{1}{96} \times r^9 = \frac{1}{96} \times \left(\frac{1}{2}\right)^9 = \frac{1}{96} \times \frac{1}{512} = \frac{1}{49152}$$

ا) $a_1 \times a_5 = a_2 \times a_4 = a_3 \times a_3 \rightarrow a_3^2 = 12 \times 96 \rightarrow a_3 = 36$

ب) $a_1 \times a_5 = a_2 \times a_4 = a_3^2 = 36 = 6^2$

$$r^a \times r^b = (r^{\sqrt{a}})^2 \rightarrow r^{a+b} = r^{\sqrt{a}} \rightarrow r^{a+b} = r^{\sqrt{a}} \rightarrow a+b = \sqrt{a}$$

$$\frac{a+b}{r} = \frac{\sqrt{a}}{r} = \sqrt{a}$$

$$a, r^2 = 1-x \quad a, r^2 = x \quad a, r^1 = x+1$$

$$\frac{a}{a, r^2} = \frac{x}{1-x} \rightarrow r^2 = \frac{x}{1-x}$$

$$\frac{a}{a, r^1} = \frac{x+1}{x} \rightarrow r^2 = \frac{x+1}{x}$$

$$\frac{x}{1-x} = \frac{x+1}{x} \rightarrow x^2 = (1-x)(x+1) \rightarrow x^2 = 1-x^2$$

$$\rightarrow 2x^2 = 1 \rightarrow x^2 = \frac{1}{2} \rightarrow x = \pm \frac{1}{\sqrt{2}}$$

$$r^2 = \frac{\frac{1}{\sqrt{2}} + 1}{\frac{1}{\sqrt{2}}} = \frac{\frac{1+\sqrt{2}}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \frac{1+\sqrt{2}}{1} = 1+\sqrt{2}$$

$$r = \sqrt{1+\sqrt{2}}$$

$a_1 + a_r = 18$ $a + ar = 18$ $a(1+r) = 18$	$a_r + a_{2r} = 18$ $ar + ar^2 = 18$ $ar(1+r) = 18$	$\frac{a(1+r^2)}{ar(1+r)} = \frac{18}{18} \rightarrow \frac{(1+r)(r^2-r+1)}{(1+r)r} = \frac{r}{r}$	6
$\rightarrow \forall r = r^2 - r + 1 \rightarrow r^2 - 10r + 9 = 0 \rightarrow r^2 - 10r + 9 = 0 \rightarrow (r-9)(r-1) = 0 \rightarrow (r-9)(r-\frac{1}{9}) = 0$ $r = 9, \frac{1}{9} \rightarrow \begin{cases} a(1+\frac{1}{9}) = 18 \rightarrow a = 16 \rightarrow 1, 9, 81, \dots \\ a(1+\frac{1}{9}) = 18 \rightarrow a = 16 \rightarrow 16, 9, 8, 1, \dots \end{cases} \textcircled{2}$			
$a_1 + ar + a_{2r} = 18$ $a + ar + ar^2 = 18$ $a + ar^2 = 10$	$a_1 \times ar \times a_{2r} = 18$ $a \times ar \times ar^2 = 18 \rightarrow a^3 r^3 = 18 \rightarrow ar = \sqrt[3]{18} \rightarrow a = \frac{\sqrt[3]{18}}{r}$	$\frac{a}{r} + \frac{ar^2}{r} = \frac{10}{r} \rightarrow 10r = r + r^3 \rightarrow r^3 - 10r + 1 = 0 \rightarrow \frac{10 \pm \sqrt{100-4}}{2} \rightarrow \begin{cases} \frac{10}{2} = 5 \rightarrow a = 1 \\ \frac{10}{2} = \frac{1}{5} \rightarrow a = 9 \end{cases}$	7
$\rightarrow \text{N.B.} : \begin{cases} r = 5 \rightarrow 1, 5, 25, \dots \\ r = \frac{1}{5} \rightarrow 9, 3, 1, \dots \end{cases}$			
$2, 8, 18, \dots \rightarrow r = 3 \rightarrow a_n = 2 \times 3^{n-1}$ ا) $2 \times \left(\frac{3^k - 1}{3 - 1} \right) = 2 \times \frac{3^k - 1}{2} = 3^k - 1$ ب) $\left\{ \begin{matrix} a_n = 2 \times 3^{n-1} \\ a_1 = 2 \end{matrix} \right\} \rightarrow (2 \times 3^9 \times 2)^{\frac{1}{10}} = (2^2 \times 3^9)^{\frac{1}{10}} = 2^{\frac{2}{10}} \times 3^{\frac{9}{10}} = 2^{\frac{1}{5}} \times 3^{\frac{9}{10}}$			8
a, ar, ar^2, ar^3 $\rightarrow ar - a = a(r-1)$ $\rightarrow ar^2 - ar = ar(r-1)$ $\rightarrow ar^3 - ar^2 = ar^2(r-1)$			9
ا) $S_n = a_1 + a_1 + d + a_1 + 2d + \dots + a_1 + (n-1)d \rightarrow na_1 (1 + 1 + \dots + n-1)d$ $= na_1 + \frac{n(n-1)}{2}d = \frac{n}{2} (2a_1 + (n-1)d)$ ب) $a + ar + ar^2 + \dots + ar^{n-1} = A$ $ar + ar^2 + ar^3 + \dots + ar^n = Ar$ $Ar - A = ar^n - a \rightarrow A(r-1) = a(r^n - 1) \rightarrow A = a \left(\frac{r^n - 1}{r - 1} \right)$			10