پرنیان دهقانی ۲۰

۱)

الف) $d=2 \quad a_{10} = \frac{1}{2} \times 2^{9} = 2^{8} = 256$

ب) $a_1 d^{4} = \frac{1}{2} \times 8 = 4 \quad a_1 d^{9} = \frac{1}{2} \times 2^{9} = 2^{8} \quad \frac{a_1 d^{4}}{a_1} = d^{4} = 16$

ج) $a_5 \quad \bigcirc \quad a_{10}$

$a_5 \to 4 \qquad a_{10} \to 256$

$b^{2} = 256 \times 4 = 1024 \to b = 32$

د) $128 = \frac{1}{2} \times d \to d^{n-1} = 2 \times 128 = 256 \to n-1 = 8 \to n = 9$

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۲) $a_5 = 12 \quad a_8 = 96 \quad \frac{a_8}{a_5} = d^{3} \quad \frac{96}{12} = 8 \to d^{3} = 8 \to d = 2$

$a_{10} = a_8 d^{2} \Rightarrow 96 \times 4 = 384$

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۳)

الف) $\overbrace{a_1 \times a_2 \times a_3 \times a_4 \times a_5}^{a_3 \times a_3} = a_3^{5} = 243 \to a_3 = \sqrt[5]{243} = 3$

($a_2 \times a_4 = a_3 \times a_3$)

ب) $a_1 \times a_5 = a_3 \times a_3 = 3 \times 3 = 9$

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۴) $2^{a+b} = (4\sqrt{2})^{2} \to 2^{a+b} = 32 = 2^{5} \to a+b = 5 \to$ واسطه حسابی $= \frac{5}{2}$

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۵) $a_3 \quad a_7 \quad a_{11}$

$1-x \quad x \quad x+1$

$b^{2} = ac$

$x^{2} = (1-x)(1+x)$

$x^{2} = 1 - x^{2} \to 2x^{2} = 1 \qquad x^{2} = \frac{1}{2}$

$x = \pm \frac{\sqrt{1}}{\sqrt{2}} \longrightarrow$ منفی آن نیز قابل قبول است چون q منفی می‌شود $\longrightarrow \pm \frac{\sqrt{1}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

۶)

$$\begin{cases} a_1 + a_1 d^3 = 28 \\ a_1 d + a_1 d^2 = 12 \end{cases}$$

$x, xd, xd^2, xd^3 = 40$

$x(1+d+d^2+d^3) = 40$, $40 = 1 \times 40$

$a_1 = 1$, $d = 3$, بزرگترین $= a_1 d^3 = 27$

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۷)

$\frac{x}{d}, x, xd \rightarrow$ ضرب $= x^3 \rightarrow 27 \rightarrow x = 3$

$\frac{3}{d}, 3, 3d \rightarrow 13 \rightarrow \frac{3 + 3d + 3d^2}{d} = 13 \rightarrow 3 + 3d + 3d^2 = 13d$

$3d^2 - 10d + 3 = 0$

$d^2 - 10d + 9 = 0 \rightarrow$ ریشه‌ها $= 3 , \frac{1}{3}$

سه عدد $\Leftarrow$ $\begin{cases} 1, 3, 9 \\ 9, 3, 1 \end{cases}$

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۸)

$a_n = 2, 6, 18, \ldots \rightarrow d = 3$

الف) $a\left(\frac{q^n - 1}{q - 1}\right) = 2\left(\frac{3^{10} - 1}{3 - 1}\right) = 3^{10} - 1$

ب)

$a_1 \times a_2 \times a_3 \times \cdots \times a_{10} = a_1 \times a_1 q \times a_1 q^2 \times \cdots \times a_1 q^9$

$(a_1)^{10} \times q^{45} = 2^{10} \times 3^{45} = 1024 \times 3^{45}$

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۹)

$A = a_1, a_2, a_3, a_4, \ldots, a_n$

$\downarrow$: $a, aq, aq^2, aq^3, \ldots, aq^n$

$a_2 - a_1 = aq - a = a(q-1)$

$a_3 - a_2 = aq^2 - aq = aq(q-1)$

$a_4 - a_3 = aq^3 - aq^2 = aq^2(q-1)$

$q = \frac{x_2}{x_1} = \frac{aq(q-1)}{a(q-1)} = q$

$A = a(q-1), aq(q-1), aq^2(q-1), \ldots$

(الف (10

$S_n = \frac{n}{2}(2a + (n-1)d)$

↳ $S_n = a_1 + (a_1+d) + (a_1+2d) + \cdots + (a_1+(n-1)d)$

$S_n = a_n + a_n - d + a_n - 2d + \cdots + a_1 + d + a_1$

$2S_n = n(2a_1 + d(n-1)) \Rightarrow S_n = \frac{n}{2}(2a_1 + d(n-1))$

ب) $S_n = a_1 + a_2 + a_3 + \cdots + a_n$

$-S_n = -a_1 - a_1 q - a_1 q^2 - \cdots - a_1 q^{n-1}$

$S_n q = a_1 q + a_1 q^2 + a_1 q^3 + \cdots + a_1 q^n$

$q S_n - S_n = a q^n - a \Rightarrow S_n = a\left(\frac{q^n - 1}{q-1}\right)$