subject

Year Month Date ( )

۲۵

آرمینا کبیری گروه دهم پنجشنبه

۱)

الف) $a_n = \frac{1}{2} \times 2^{n-1} = 2^{n-2}$, $a_{10} = 2^{10-2} = 2^8 = 256$

ب) $\frac{a_{17}}{a_{13}} = \frac{aq^{16}}{aq^{12}} = q^4 = 2^4 = 16$

ج) $\sqrt{a_4 a_{10}} = \sqrt{a_1 q^{12}} = aq^6 = \frac{1}{2} \times 2^6 = 32$: واسطه هندسی ۴ و ۱۰

د) نهمین جمله $2^{n-2} = 128 = 2^7 \to n-2 = 7 \to n = 9$

۲)

$\frac{a_8}{a_5} = \frac{aq^7}{aq^4} = q^3 = \frac{96}{12} = 8 \to q = 2$

$aq^4 = 12 \to a = \frac{12}{16} = \frac{3}{4}$, $aq^9 = \frac{3}{4} \times 2^9 = 384$

$a_{10} = 384$

۳) الف) $a_1 \times a_5 = a_2 \times a_4 = a_3 \times a_3 \to a_3^5 = 243 \to a_3 = 3$

ب) $a_1 \times a_5 = a_3^2 = 3^2 = 9$

۴) $2^a \times 2^b = (4\sqrt{2})^2 = 32 \to 2^a \times 2^b = 2^5 \to a+b = 5$

واسطه حسابی a, b : $\frac{5}{2}$

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$$x^2 = (x+1)(1-x) \rightarrow x^2 = x - x^2 + 1 - x \rightarrow 2x^2 = 1 \rightarrow x = \pm \frac{\sqrt{2}}{2}$$

منفی قابل قبول نیست:

$$\frac{a_7}{a_{11}} = q^4 = \frac{-\frac{\sqrt{2}}{2} + 1}{-\frac{\sqrt{2}}{2}} = \frac{-\sqrt{2}+2}{-\sqrt{2}} = \frac{(-\sqrt{2}+2)(-\sqrt{2})}{2}$$

$$= \frac{2-2\sqrt{2}}{2} = 1-\sqrt{2} < 0 \rightarrow q^4 < 0$$ غلط

$$\rightarrow x = +\frac{\sqrt{2}}{2}$$

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$$a+a_4=28 \rightarrow a+aq^3=28$$
$$a_2+a_3=12 \rightarrow aq^2+aq=12$$
$$\rightarrow \frac{a(1+q^3)}{aq(1+q)}=\frac{\cancel{a}\cancel{(1+q)}(1-q+q^2)}{\cancel{a}q\cancel{(1+q)}}=\frac{7}{3}$$

$$\rightarrow 7q=3-3q+3q^2 \rightarrow 3q^2-10q+3=0 \rightarrow (3q-1)(q-3)=0$$

$$\rightarrow q=\frac{1}{3},\ a=27 \rightarrow 27,9,3,1$$

$$q=3,\ a=1 \rightarrow 1,3,9,27$$

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$$\frac{x}{q},\ x,\ xq \rightarrow x^3=27 \quad x=3$$

$$\frac{3}{q}+3+3q=13 \rightarrow \frac{3+3q^2}{q}=10 \rightarrow 10q=3+3q^2 \rightarrow 3q^2-10q+3=0$$

$$\rightarrow (3q-1)(q-3)=0 \rightarrow q=\frac{1}{3} \rightarrow 3x+x+\frac{1}{3}x=13 \rightarrow \frac{13}{3}x=13 \rightarrow x=3$$

$$\rightarrow \boxed{9,3,1}$$

$$q=3 \rightarrow \frac{x}{3},x,3x=13 \rightarrow x=3 \rightarrow \boxed{1,3,9}$$

Parsian

۸) الف) $(3^{10}-1) = S_{10} = 2\,\frac{3^{10}-1}{3-1}$ زیرا $S_n = a\left(\frac{q^n-1}{q-1}\right)$

ب) $(2^{10}\times 3^{45}) = (2^2\times 3^9)^5 = (a_1\times a_{10})^5 = \sqrt{(a_1\times a_{10})^{10}} \leftarrow a_{10} = 2\times 3^9$

۹) قدر نسبت همان q

$$a \xrightarrow{\times q} aq,\ aq^2,\ \dots$$

$$(aq-a),\ (aq^2-aq),\ (aq^3-aq^2),\ \dots$$

$$a(q-1) \xrightarrow{\times q} aq(q-1),\ aq^2(q-1)$$

$q = \frac{aq(q-1)}{a(q-1)}$: قدر نسبت

۱۰) الف)

$$a_1+a_2+\dots+a_n \to a_1+a_n = a_2+a_{n-1} = a_3+a_{n-2} = \dots$$

تعداد $\frac{n}{2}$

$$S_n = \frac{n}{2}(a_1+a_n) = \frac{n}{2}(a_1+a_1+(n-1)d)$$
$$= \frac{n}{2}(2a_1+(n-1)d)$$

ب)

$$S = a+aq+aq^2+\dots+aq^{n-1}$$
$$\xrightarrow{\times q} qS = aq+aq^2+\dots+aq^{n-1}+aq^n$$
$$qS-S = aq^n-a \to S(q-1) = a(q^n-1) \to S = a\,\frac{q^n-1}{q-1}$$

روش دوم الف)

$$\text{مجموع} = \frac{\text{تعداد جملات}\,(\text{آخری}+\text{اولی})}{2} = \frac{n(a_1+a_n)}{2} = \frac{n}{2}(a_1+a_1+(n-1)d)$$
$$= \frac{n}{2}(2a_1+(n-1)d)$$