نام: رفیعی — گروه دهم پنجشنبه (دختر) — تکلیف شماره ۱۴

۱- الف) $t_n = \frac{1}{2} \times (2)^{n-1} \to t_{10} = \frac{1}{\cancel{2}} \times \cancel{2}^{9} = 2^{8} = 256$

ب) $\frac{a_{17}}{a_{13}} = \frac{ar^{16}}{ar^{12}} = r^4 \Rightarrow 2^4 = 16$

ج) $\sqrt{a_4 a_{10}} = \sqrt{a^2 r^{12}} = ar^6 = \frac{1}{\cancel{2}} \times (\cancel{2})^{6} = 2^5 = 32$

د) $128 = ar^{n-1} \Rightarrow 128 = \frac{1}{2} \times 2^{n-1} \to 256 = 2^{n-1} \to 8 = n-1$

$n = 9$ جمله‌ی نهم

۲- $ar^4 = 12$, $ar^7 = 96$

$r^3 = \frac{ar^7}{ar^4} = \frac{96}{12} = 8 \to r^3 = 8 \to r = 2$

$ar^4 = 12 \to a = \frac{12}{16} = \frac{3}{4}$

$ar^9 = \frac{3}{\cancel{4}} \times \cancel{2}^{9} = 3 \times 128 = 384$

۳- الف) $\underbrace{a_1 \times a_5}_{a_3 \times a_3} = \underbrace{a_2 \times a_4}_{a_3 \times a_3} = a_3 \times a_3 \to (a_3)^5 = 243 \to a_3 = 3$

ب) $a_1 \times a_5 = (a_3)^2 = (3)^2 = 9$

۴- $2^a \times 2^b = (4\sqrt{2})^2 \to 2^a \times 2^b = 32 = 2^5 \to a+b = 5$ واسطه‌ی حسابی: $\frac{5}{2} = 2.5$

۵- $\underbrace{(x+1)(1-x)}_{\cancel{x} - x^2 + 1 - \cancel{x} = -x^2+1} = x^2 \to x^2 = 1 - x^2 \Rightarrow 2x^2 = 1 \to x^2 = \frac{1}{2} \Rightarrow x = \pm\sqrt{\frac{1}{2}}$

$x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$ $\qquad \frac{a_7}{a_3} = r^4 \to \frac{x}{1-x}$

$\frac{-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}} = \frac{-\sqrt{2}}{2+\sqrt{2}} = \frac{(-\sqrt{2})(2-\sqrt{2})}{\underbrace{(2+\sqrt{2})(2-\sqrt{2})}_{4-2}} = \frac{-2\sqrt{2}+2}{2} = -\sqrt{2}+1 \to 1-\sqrt{2} < 0 \to r^4 < 0$ غ ق ق

جواب: $x = +\frac{\sqrt{2}}{2}$

۶-

$a_1+a_4=28 \rightarrow a+ar^3=28 \rightarrow a(1+r^3)$

$a_2+a_3=12 \rightarrow ar+ar^2=12 \rightarrow ar(1+r)$

$\Big\} \dfrac{\overbrace{a(\cancel{1+r})(1+r^2-r)}^{\text{فیل و فنجان}}}{ar(\cancel{1+r})} = \dfrac{\cancel{28}\,7}{\cancel{12}\,3}$

$7r = 3+3r^2-3r \rightarrow \underbrace{3r^2-10r+3=0}_{\times 3} \rightarrow (\underbrace{3r-1}_{\frac{1}{3}})(\underbrace{r-3}_{3})=0$

$\rightarrow r^2-10r+9 \rightarrow \dfrac{(\underbrace{r-1}_{1})(\underbrace{r-9}_{9})}{3} \Big\} \dfrac{1}{3}, \dfrac{9}{3}=3$

if $r=\frac{1}{3} \rightarrow a=27 \rightarrow 27, 9, 3, 1$

if $r=3 \rightarrow a=1 \rightarrow 1, 3, 9, 27$

$\Rightarrow$ بزرگترین عدد: 27

۷-

$\dfrac{a}{r}, a, ar \rightarrow a^3=27 \rightarrow a=3 \qquad \dfrac{3}{r}+3+3r=13$

$\dfrac{a}{r}+ar+a \rightarrow \dfrac{3}{r}+3+3r=13 \rightarrow \dfrac{3}{r}+3r=10 \rightarrow \dfrac{3+3r^2}{r}=10$

$10r=3+3r^2 \rightarrow 3r^2-10r+3=0 \rightarrow r^2-10r+9 \rightarrow \dfrac{(\underbrace{r-1}_{1})(\underbrace{r-9}_{9})}{3} \Big\} \dfrac{1}{3}, 3$

if $r=3 \rightarrow 1, 3, 9$ $\qquad$ if $r=\frac{1}{3} \rightarrow 9, 3, 1$

۸- الف)

$S_n = a\left(\dfrac{q^n-1}{q-1}\right) \Rightarrow S_{10} = \dfrac{2(3^{10}-1)}{3-1} = 3^{10}-1$

ب)

$a_{10} = 2\times 3^9 \qquad \sqrt{(\text{جمله اول} \times \text{آخر})^n} \Rightarrow \sqrt{(a_1\times a_{10})^{10}} = (\underbrace{a_1\times a_{10}}_{2^2\times 3^9})^5 = 2^{10}\times 3^{45}$

۹-

$\underbrace{a, ar}_{\times r}, \underbrace{ar^2}_{\times r}, \ldots \rightarrow \underbrace{(ar-a)}_{a(r-1)}, \underbrace{(ar^2-ar)}_{ar(r-1)}, \underbrace{(ar^3-ar^2)}_{ar^2(r-1)}, \ldots$

$a(r-1) \xrightarrow{\times r} ar(r-1) \xrightarrow{\times r} ar^2(r-1)$

قدر نسبت $\Rightarrow \dfrac{\cancel{a}r^{\cancel{2}}(\cancel{r-1})}{\cancel{a}\cancel{r}(\cancel{r-1})} = r$

۱۰- الف)

$a_1+a_2+a_3+\ldots+a_n \rightarrow \underbrace{a_1+a_n = a_2+a_{n-1} = a_3+a_{n-2} = \ldots}_{\text{تعداد} = \frac{n}{2}}$

$S_n = \dfrac{n}{2}(a+L) \rightarrow S_n = \dfrac{n}{2}(a_1+a_n) \rightarrow \dfrac{n}{2}(a_1+a_1+(n-1)d) \Rightarrow \dfrac{n}{2}(2a+(n-1)d)$

$a_n = a_1+(n-1)d$

ب)

$\times r \Big(S = a+ar+ar^2+\ldots+ar^{n-1}$

$\Big(rS = ar+ar^2+ar^3+\ldots+ar^n$

$rS-S = ar^n-a \rightarrow S(r-1) = a(r^n-1) \rightarrow S = \dfrac{a(r^n-1)}{r-1}$