۲۰

نگار فتحی — گروه دهم پنجشنبه (دختر) — تکلیف شماره ۱۴

۱- الف) $t_n = \frac{1}{2}\times(2)^{n-1} \to t_{10} = \frac{1}{\cancel{2}}\times \cancel{2}^{9} = 2^{8} = 256$

۵

ب) $\frac{a_{17}}{a_{13}} = \frac{a\cancel{r^{16}}}{a\cancel{r^{12}}} = r^4 \Rightarrow 2^4 = 16$

ج) $\sqrt{a_4 a_{10}} = \sqrt{a^2 r^{12}} = ar^6 = \frac{1}{\cancel{2}}\times(\cancel{2})^{6} = 2^5 = 32$

د) $128 = ar^{n-1} \Rightarrow 128 = \frac{1}{2}\times 2^{n-1} \to 256 = 2^{n-1} \to 8 = n-1$

$n = 9$ جمله‌ی نهم

۲- $\begin{cases} ar^4 = 12 \\ ar^7 = 96 \end{cases}$

$r^3 = \frac{\cancel{a}r^7}{\cancel{a}r^4} = \frac{96}{12} = 8 \to r^3 = 8 \to r = 2$

$ar^4 = 12 \to a = \frac{12}{16} = \frac{3}{4}$

$ar^9 = \frac{3}{\cancel{4}}\times \cancel{2}^{9} = 128 \to 384$

۵

۳- الف) $\underbrace{a_1\times a_5}_{a_3\times a_3} = \underbrace{a_2\times a_4}_{a_3\times a_3} = a_3\times a_3 \to (a_3)^5 = 243 \to a_3 = 3$

۵

ب) $a_1\times a_5 = (a_3)^2 = (3)^2 = 9$

۴- $2^a\times 2^b = (4\sqrt{2})^2 \to 2^a\times 2^b = \cancel{32} = 2^5 \to a+b = 5$ ۵

واسطه‌ی حسابی: $\frac{5}{2} = 2/5$

۵- $(x+1)(1-x) = x^2 \to x^2 = 1-x^2 \Rightarrow 2x^2 = 1 \to x^2 = \frac{1}{2} \Rightarrow x = \pm\sqrt{\frac{1}{2}}$ ۵

$\cancel{x - x^2 + 1 - x} = -x^2+1$

$x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$\frac{a_7}{a_3} = r^4 \to \frac{x}{1-x}$

$\frac{-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}} = \frac{-\sqrt{2}}{2+\sqrt{2}} = \frac{(-\sqrt{2})(2-\sqrt{2})}{\cancel{(2+\sqrt{2})(2-\sqrt{2})}} = \frac{-\cancel{2}\sqrt{2}+1}{\cancel{2}} = -\sqrt{2}+1 \to 1-\sqrt{2} < 0$

$4-2=2$

$r^4 < 0$ غ ق ق

جواب: $x = +\frac{\sqrt{2}}{2}$

-6

فیل و فنجان

$a_1+a_4=28 \to a+ar^3=28 \to a(1+r^3)$ $\Big\}\ \dfrac{\cancel{a}\cancel{(1+r)}(1+r^2-r)}{\cancel{a}r\cancel{(1+r)}} = \dfrac{\cancel{28}^{7}}{\cancel{12}_{3}}$

$a_2+a_3=12 \to ar+ar^2=12 \to ar(1+r)$

$7r = 3+3r^2-3r \to \underbrace{3r^2-10r+3=0}_{\div 1} \to (\underbrace{3r-1}_{\frac13})(\underbrace{r-3}_{3})=0$

$\to r^2-10r+9 \to (\underbrace{r-1}_{1})(\underbrace{r-9}_{9})\ \Big\}\ \frac13,\ \frac93=3$

if $r=\frac13 \to a=27 \to 27, 9, 3, 1$

if $r=3 \to a=1 \to 1, 3, 9, 27$

$\Rightarrow$ بزرگترین عدد: 27

-7

$\frac{a}{r}, a, ar \to a^3=27 \to a=3$ $\qquad \frac{3}{r}+3+3r=13$

$\frac{a}{r}+ar+a \to \frac{3}{r}+3+3r=13 \to \frac{3}{r}+3r=10 \to \frac{3+3r^2}{r}=10$

$10r=3+3r^2 \to 3r^2-10r+3=0 \to r^2-10r+9 \to (\underbrace{r-1}_{1})(\underbrace{r-9}_{9})\ \Big\}\ \frac13,\ 3$

if $r=3 \to 1, 3, 9$ $\qquad$ if $r=\frac13 \to 9, 3, 1$

-8 الف)

$S_n = a\left(\frac{q^n-1}{q-1}\right) \Rightarrow S_{10} = \frac{\cancel{2}(3^{10}-1)}{\underbrace{3-1}_{\cancel{2}}} = 3^{10}-1$

ب)

$a_{10}=2\times 3^9$ $\qquad \sqrt{(\text{جمله اول} \times \text{آخر})^n} \Rightarrow \sqrt{(a_1\times a_{10})^{10}} = (\underbrace{a_1\times a_{10}}_{2^2\times 3^9})^5 = 2^{10}\times 3^{45}$

-9

$a, ar, ar^2, \ldots \to (ar-a), (ar^2-ar), (ar^3-ar^2), \ldots$

$\underbrace{(ar-a)}_{a(r-1)}, \underbrace{(ar^2-ar)}_{ar(r-1)}, \underbrace{(ar^3-ar^2)}_{ar^2(r-1)}$ $\qquad (\times r)$

$\Rightarrow$ قدر نسبت $\dfrac{\cancel{a}r^{\cancel{2}}\cancel{(r-1)}}{\cancel{a}\cancel{r}\cancel{(r-1)}} = r$

-10 الف)

$a_1+a_2+a_3+\ldots+a_n \to \underbrace{a_1+a_n = a_2+a_{n-1} = a_3+a_{n-2} = \ldots}_{\text{تعداد} = \frac{n}{2}}$

$S_n = \frac{n}{2}(a+L) \to S_n = \frac{n}{2}(a_1+a_n) \to \frac{n}{2}(a_1+a_1+(n-1)d) \Rightarrow \frac{n}{2}(2a+(n-1)d)$

$a_n = a_1+(n-1)d$

ب)

$\times r\ (S = a+ar+ar^2+\ldots+ar^{n-1})$

$rS = ar+ar^2+ar^3+\ldots+ar^n$

$rS-S = ar^n-a \to S(r-1)=a(r^n-1) \to S=\dfrac{a(r^n-1)}{r-1}$