مهرانا فراغتی - دهم دختر C - تکلیف شماره ۱۴

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①

ص

$a_n = \frac{1}{2}, 1, 2$

$\times 2 \quad \times 2 \to r = 2$

$a_n = a_1 \times r^{(n-1)}$

الف) $a_{10} = a_1 \times r^{(10-1)}$

$a_{10} = \frac{1}{2} \times 2^9 = 2^{-1} \times 2^9 = 2^8 \to \boxed{a_{10} = 2^8 = 256}$

ب) $\frac{a_{17}}{a_{13}} = \frac{a_1 \times r^{16}}{a_1 \times r^{12}} = \frac{r^{16}}{r^{12}} = r^{16} \times r^{-12} = r^4$

$r^4 = 2^4 = 16 \to \boxed{\frac{a_{17}}{a_{13}} = 16}$

ج) $\frac{a_{10}}{a_4} = (r')^2 \to \frac{a_4 r^6}{a_4} = (r')^2 \to r^6 = (r')^2$

$r^6 = (r')^2 \to (r^3)^2 = (r')^2 \to r^3 = r' \to r' = 2^3 = 8$

$a_4 = a_1 \times r^3 = \frac{1}{2} \times 2^3 = 2^{-1} \times 2^3 = 2^2 = 4$

$a_4 \times r' = 4 \times 8 = \boxed{32}$

د) $a_n = a_1 \times r^{(n-1)} \to a_n = \frac{1}{2} \times 2^{(n-1)} = 128$

$128 = 2^{-1} \times 2^{n-1} = 2^{n-2} \to 2^7 = 2^{n-2}$

$7 = n - 2 \to \boxed{n = 9}$

②

ص

$a_5 = 12$

$a_8 = 96$

$\frac{a_8}{a_5} = \frac{a_5 r^3}{a_5} = r^3 \to \frac{96}{12} = r^3 \to 8 = r^3 \to 2^3 = r^3$

$\to r = 2$

$a_{10} = a_5 r^5 = 12 \times 2^5 = 12 \times 32 = \boxed{384}$

③

$a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 243$

الف) $\frac{a_3}{r^2} \times \frac{a_3}{r} \times a_3 \times a_3 r \times a_3 r^2 = (a_3)^5$

ص

$(a_3)^5 = 243 \to (a_3)^5 = 3^5 \to \boxed{a_3 = 3}$

ب) $a_1 \times a_5 = ?$

$a_1 \times a_5 = a_3 \times a_3 \to a \times ar^4 = ar^2 \times ar^2 \to a^2 r^4 = a^2 r^4$

$a_1 \times a_5 = (a_3)^2 = (3)^2 = 9 \to \boxed{a_1 \times a_5 = 9}$

④

$2^a, 4\sqrt{2}, 2^b$

$\frac{2^b}{4\sqrt{2}} = \frac{4\sqrt{2}}{2^a} \to 2^b \times 2^a = (4\sqrt{2})^2 \to 2^{a+b} = 32$

ص

$2^{a+b} = 32 = 2^5 \to a + b = 5$

$\frac{a+b}{2} = \frac{5}{2} = \boxed{2.5}$

⑤

$a_3 = 1 - x$

$a_7 = x$

$a_{11} = x + 1$

$\frac{a_{11}}{a_7} = \frac{a_7}{a_3} \to \frac{a_7 r^4}{a_7} = \frac{a_3 r^4}{a_3} \to r^4 = r^4$

$\frac{x+1}{x} = \frac{x}{1-x} \to x^2 = (x+1)(1-x)$

$x^2 = x - x^2 + 1 - x \to x^2 = -x^2 + 1 \to 2x^2 = 1 \to x^2 = \frac{1}{2}$

$x^2 = \frac{1}{2} \to x = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

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$\boxed{x = \pm\frac{\sqrt{2}}{2}}$ → فقط $\frac{\sqrt{2}}{2}$ قابل قبول

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$a_1 + a_4 = 28 \rightarrow a + ar^3 = 28$

$a_2 + a_3 = 12 \rightarrow ar + ar^2 = 12$

$\frac{a(1+r^3)}{a(r+r^2)} = \frac{28}{12} \rightarrow \frac{1+r^3}{r+r^2} = \frac{7}{3} \rightarrow \frac{\cancel{(r+1)}(r^2-r+1)}{r\cancel{(1+r)}} = \frac{7}{3}$

$\frac{r^2-r+1}{r} = \frac{7}{3} \rightarrow 3r^2 - 3r + 3 = 7r \rightarrow 3r^2 - 10r + 3 = 0$

$r^2 - 10r + 9 = 0 \rightarrow (r-1)(r-9) = 0 \rightarrow r = \frac{1}{3}, 3$

$a + ar^3 = 28a = 28 \rightarrow a = 1 \rightarrow r = 3 \rightarrow 1, 3, 9, 27$

$a + ar^3 = \frac{28}{27}a = 28 \rightarrow a = 27 \rightarrow r = \frac{1}{3} \rightarrow 27, 9, 3, 1$ $\Big\} = \boxed{27}$

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$a_1 + a_2 + a_3 = 13$

$a_1 \times a_2 \times a_3 = 27$

$\frac{a_2}{r} \times a_2 \times a_2 r = (a_2)^3 = 27$

$(a_2)^3 = 3^3 \rightarrow a_2 = 3$

$\frac{a_2}{r} + a_2 + a_2 r = 13$

$a_2\left(\frac{1}{r} + 1 + r\right) = 3\left(\frac{1}{r} + 1 + r\right) = 13$

$\frac{1}{r} + 1 + r = \frac{13}{3} \rightarrow \frac{1}{r} + \frac{r}{r} + \frac{r^2}{r} = \frac{13}{3} \rightarrow \frac{1+r+r^2}{r} = \frac{13}{3}$

$3 + 3r + 3r^2 = 13r \rightarrow 3r^2 - 10r + 3 = 0 \rightarrow r^2 - 10r + 9 = 0$

$(r-1)(r-9) = 0 \rightarrow r = \frac{1}{3}, \frac{9}{3} \rightarrow r = \frac{1}{3}, 3$

$r = 3 \rightarrow \boxed{1, 3, 9}$

$r = \frac{1}{3} \rightarrow \boxed{9, 3, 1}$

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$a_n = 2 \xrightarrow{\times 3} 6 \xrightarrow{\times 3} 18 \rightarrow q = 3$

الف) $S_{10} = a_1 \left(\frac{q^n - 1}{q - 1}\right)$

$S_{10} = 2\left(\frac{3^{10}-1}{3-1}\right) = \cancel{2} \times \frac{3^{10}-1}{\cancel{2}} = 3^{10} - 1 = 59049 - 1 =$

$\boxed{S_{10} = 59048} = \boxed{3^{10} - 1}$

ب) $a_1 \times a_2 \times a_3 \times \dots \times a_{10} =$

$(a_1)^{10} \times r^{(1+2+\dots+9)} = (a_1)^{10} \times r^{\frac{9\times 10}{2}} = (a_1)^{10} \times r^{45}$

$= (2)^{10} \times (3)^{45} = \boxed{2^{10} \times 3^{45}}$

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$a_{n+1} - a_n = a_n r - a_n = a_n(r-1)$

$a_{n+2} - a_{n+1} = a_n r^2 - a_n r = a_n \times r(r-1)$

$a_{n+3} - a_{n+2} = a_n r^3 - a_n r^2 = a_n \times r^2(r-1)$

$a_n \times (r-1) \xrightarrow{\times r} a_n \times r \times (r-1) \xrightarrow{\times r} a_n \times r^2 \times (r-1)$ → تصاعد هندسی

$\boxed{\text{قدر نسبت} = r = q}$

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$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$a_1 + a_2 + a_3 + \dots + a_n$$

$$a + a + d + a + 2d + \dots + a + (n-1)d$$

$$= na + (1 + 2 + \dots + (n-1))d$$

$$na + \left[\frac{(n-1+1)(n-1)}{2}\right]d = na + \left[\frac{n(n-1)}{2}\right]d$$

$$na + \frac{n(n-1)}{2}d = \boxed{\frac{n}{2}(2a + (n-1)d)}$$

$$S_n = a\left(\frac{q^n - 1}{q-1}\right)$$

$$S_n = a_1 + a_2 + a_3 + \dots + a_n$$

$$S_n = a + aq + aq^2 + \dots + aq^{n-1}$$

$$qS_n = \cancel{aq} + \cancel{aq^2} + \cancel{aq^3} + \dots + aq^n$$

$$-S_n = -a - \cancel{aq} - \cancel{aq^2} - \dots - \cancel{aq^{n-1}}$$

$$(q-1)S_n = aq^n - a$$

$$(q-1)S_n = a(q^n - 1)$$

$$\boxed{S_n = a\left(\frac{q^n - 1}{q-1}\right)}$$