

$$a_1 + 3d = 41, a_1 = 15 \rightarrow 3d = 26 \Rightarrow d = \frac{26}{3}$$

$$a_n = 15 + (n-1) \cdot \frac{26}{3}$$

$$15 \cdot n = 15 + (n-1) \cdot \frac{26}{3} \Rightarrow 15n = 15 + \frac{26n - 26}{3} \rightarrow n = \frac{15 \cdot 3}{3} = \frac{45}{3} = 15$$

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$$100 \leq V_n + 3 \cdot 999$$

$$99 \leq V_n \leq 999$$

$$100 \leq n \leq 1000$$

$$100 \leq n \leq 1000 \rightarrow \frac{1000-100}{1} + 1 = 901$$

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$$n=0 \rightarrow a_1 + a_2 + a_3 = 12 \Rightarrow 3a_1 + 3d = 12 \Rightarrow a_1 + d = 4 \rightarrow a_2 = 4$$

$$n=3 \rightarrow a_1 + a_2 + a_3 = -4 \Rightarrow 3a_1 + 6d = -4 \Rightarrow a_1 + 2d = -\frac{4}{3} \rightarrow a_2 = -\frac{4}{3}$$

$$d = \frac{a_n - a_m}{n - m} = \frac{a_2 - a_1}{2 - 0} = \frac{-\frac{4}{3} - 4}{2} = \frac{-\frac{16}{3}}{2} = -\frac{8}{3} \Rightarrow a_1 = 4$$

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$$\begin{cases} a_{14} = a_1 + 13d \rightarrow 4 + 13 \cdot \frac{8}{3} = \frac{100}{3} \\ a_n = a_1 + (n-1)d \rightarrow 4 + (n-1) \cdot \frac{8}{3} = \frac{100}{3} \end{cases} \rightarrow a_n + a_{14} = \frac{200}{3}$$

$$\begin{cases} a_r = r^x - 1 \\ a_n = r^x + 1 \\ a_{13} = r^x - 3 \end{cases} \rightarrow \begin{cases} r^x = y \\ a_r = y - 1 \\ a_n = y + 1 \\ a_{13} = y - 3 \end{cases}$$

$$\begin{aligned} a_n - a_r &= a_{13} - a_n = ad \\ r^y - y + 1 &= y^2 - 3 - r^y \Rightarrow y^2 - 4y - 4 = 0 \end{aligned}$$

$$a_r + a_n + a_{13} = y - 1 + y + 1 + y - 3 = 3y - 3 = 3 \Rightarrow y = 2 \Rightarrow r^x = 2 \Rightarrow r = 2, x = 1$$

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$$a_{14} - a_{10} = 4d = 20 \Rightarrow d = 5$$

$$a_1 + 9d + a_1 + 11d = 20 \Rightarrow 2a_1 + 20 = 20 \Rightarrow a_1 = 0 \rightarrow a_n = 0 + (n-1) \cdot 5 = 5(n-1)$$

$$a_{11} = 5(11-1) = 50$$

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$$a_1 + ar + ar^2 + ar^3 + ar^4 = \frac{1}{r}(ar + ar^2 + ar^3 + ar^4) \Rightarrow a_1 + rd = \frac{1}{r}(ar + r^2d)$$

$$\Rightarrow r(a_1 + rd) = \frac{a}{r}(a_1 + rd) \Rightarrow a_1 + rd = \frac{a_1}{r} + \frac{r}{r}d \Rightarrow a_1 - \frac{a_1}{r} = r\frac{1}{r}d - rd \Rightarrow \frac{ra_1}{r} = \frac{d}{r}$$

$$\Rightarrow 4a_1 = rd \Rightarrow \underline{a_1 = \frac{1}{4}d}$$

$$\frac{ar}{a_1} = \frac{a_1 + d}{a_1} = \frac{\frac{1}{4}d + d}{\frac{1}{4}d} = \frac{\frac{5}{4}d}{\frac{1}{4}d} = \frac{5 \times r}{r \times 1} = \underline{5}$$

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$$t_n - t_{n-r} = rd \Rightarrow rd = 12 \Rightarrow d = 6$$

$$\frac{t_1^r - t_0^r}{tr} \Rightarrow \frac{(t_1 + t_0)(t_1 - t_0)}{t_1 + d} \Rightarrow \frac{(t_1 + rd + t_1 - d)(rd)}{t_1 + d} \Rightarrow \frac{(r + r)(rd)}{t_1 + d}$$

$$\Rightarrow \frac{(r + r)(r_0)}{t_1 + d} = \frac{r_0(t_1 + d)}{t_1 + d} = \underline{r_0}$$

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$$d = \frac{t_m - t_n}{\frac{m}{n} + 1} \Rightarrow d = \frac{r + \sqrt{r} - r + 1r\sqrt{r}}{1r} = \frac{1r\sqrt{r}}{1r} = \underline{\sqrt{r}}$$

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$$\frac{r}{\text{جواب } r_n} \frac{r}{r} \rightarrow r_{n-1} \text{ نه}$$

$$a_{n-1} = rr$$

$$\rightarrow a_{n-1} = r + (r_{n-1} - r)d = rr, d = \frac{r}{r_{n-1} - r}$$

$$a_{n+1} = 11 \rightarrow a_{n-1} - a_{n+1} = r1$$

$$(r_{n-1} - r)d = r1$$

$$d \text{ جواب } (r_{n-1} - r) \left(\frac{r}{r_{n-1} - r} \right) = r1 \rightarrow \frac{r \cdot n - r_0}{r_{n-1} - r} = r1 \rightarrow \frac{r(n-1)}{r(n-1)} = r1$$

$$r_{n-1} - r1 = r_{n-1} - r_0 \rightarrow r_{n-1} = 9$$

$$\rightarrow n = 9$$

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$$a_n + a_{n+r} = a_r + a_{1r} \rightarrow a_1 + (n-1)d + a_1 + (n+r)d = a_1 + rd + a_1 + 1rd$$

$$\Rightarrow \cancel{ra_1} + rnd + rd = \cancel{ra_1} + 1rd \Rightarrow rd(n+1) = \cancel{rd} \Rightarrow n+1 = 9 \Rightarrow \underline{n=8}$$

$$a_n = r a_{\frac{n}{r}} \Rightarrow a_1 + (n-1)d = r[a_1 + (\frac{n}{r} - 1)d] \Rightarrow a_1 + nd - d = ra_1 + \frac{rn}{r}d - rd$$

$$\Rightarrow a_1 + nd - d - ra_1 - \frac{rn}{r}d + rd = 0 \Rightarrow -ra_1 + \frac{-n+r}{r}d = 0 \rightarrow (-n+r)d = ra_1 \xrightarrow{n=1} -d = ra_1 \rightarrow a_1 = \underline{\frac{-d}{r}} \text{ ①}$$

$$a_k = 0 \rightarrow a_k = a_1 + (k-1)d \xrightarrow{\text{①}} -d + (k-1)d = rd(k-r) = 0 \begin{cases} d=0 \rightarrow \text{جواب } 0 \\ k-r=0 \rightarrow k=r \rightarrow a_r = \underline{\frac{-d}{r}} \end{cases}$$