دهم پنجشنبه — ریحانه مجیبی

۱) الف) $t_n = 5, 9, 13, 17, \dots$ ($+4$ $+4$ $+4$)

$t_n = 4n+1$

ب) $t_n = 4n+1 \to t_{10} = 4\times10+1 = 40+1 = 41$

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۲) الف) $t_n = 6, 10, 14, 18, \dots$

$S_n = \frac{n}{2}(2a_1+(n-1)d)$

$S_{10} = \frac{10}{2}(2\times6+(10-1)\times4) \to 5\times(36+12) = 5\times48 = 240$

ب) $4\times8 = 32 \to$ یعنی آخرین عدد این ۵ جمله t_{32} است

$t_{28}+t_{29}+t_{30}+t_{31}+t_{32} =$

$t_{30}-2d+t_{30}-d+t_{30}+t_{30}+d+t_{30}+2d = 5t_{30}$

$t_{30} = t_1+29d \to t_{30} = 6+(29\times4) \Rightarrow t_{30} = 122$ (116)

$5t_{30} \to 5\times122 = 610$

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۳) $a_1 = 1+\sqrt3$, $a_2 = 2$, $a_3 = 3-\sqrt3$

چون دنباله حسابی است پس تفاضل بین جملات برابر ثابت است $\Rightarrow a_2-a_1 = a_3-a_2 = d$

$d = a_2-a_1 = 2-(1+\sqrt3) = 1-\sqrt3$

$a_n = a_1+(n-1)d \to a_{25}-a_{23} = (a_1+24d)-(a_1+22d) = 2d$

$a_{25}-a_{23} = 2(1-\sqrt3) = 2-2\sqrt3$

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۴) $a_n = 5^{2x}, 3\times25^x, 5^y \to 3\times25^x = 3\times5^{2x}$

$a_2-a_1 = a_3-a_2$

$3\times5^{2x}-5^{2x} = 5^y-3\times5^{2x}$

$2\times5^{2x} = 5^y-3\times5^{2x}$

$2\times5^{2x}+3\times5^{2x} = 5^y$

$5\times5^{2x} = 5^y$

$5^{1+2x} = 5^y$

$y = 2x+1$ ★

$b_n = x, 2, y$

$b_2-b_1 = b_3-b_2$

$2-x = y-2$

$2+2 = x+y$

$x+y = 4$ ★

★ $\to y = 2x+1$

جایگذاری $x+y=4 \to x+(2x+1) = 4$

$3x+1 = 4$

$3x = 3 \to x = 1$

$y = 3$

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۵) $a_n = 2x-4, 2x-1, 6x, \dots$

$d = a_2-a_1 \to d = (2x-1)-(2x-4)$

$d = 2x-1-2x+4 \to d = 3$

پیدا کردن x مقدار $\to d = a_3-a_2$

$3 = 6x-(2x-1)$

$3 = 6x-2x+1$

$3 = 4x+1$

$2 = 4x$

$x = \frac{1}{2}$

$a_4 = a_3+d$

$a_4 = 6x+3 \to a_4 = (6\times\frac{1}{2})+3 = 6$

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۶) $a_n = 3, 5, 7, \dots \to t_n = 2n+1$ ($+2$ $+2$)

$t_{20} = 2\times20+1 = 41$

$b_n = 2, 5, 8, \dots \to t_n = 3n-1$ ($+3$ $+3$)

$t_{20} = 3\times20-1 = 59$

$+6$ $+6$

$3, (5), 7, 9, (11), 13, 15, (17), \dots 41$

$2, (5), 8, (11), 14, (17), \dots 59$

مشترک $d = 6 \to a_n = 6n-1 \to n = 7$

$a_7 = 6\times7-1 = 41$

(۷)

$a_1+a_2+a_3=21$

$a_2+a_3+a_4=21 \rightarrow 3a_3=21 \rightarrow a_3=7$

$-14, 7, 35$

$a_2-a_3+a_1=35-7-21=7$

$a_4+a_5+a_6=105$

$a_5+a_6+a_7=147$

$3a_6=147 \rightarrow a_6=49$

$\frac{7}{a_1}=\frac{7}{-21}=\boxed{-\frac{1}{3}}$

(۸)

$a_1+a_2+a_3=15$

$a_2+a_3+a_4=30 \rightarrow 3a_3=30 \rightarrow a_3=10$

$2, 5, 8, 11$

$+3 \quad +3 \quad +3$

$a_{10}=a_1+(n-1)d \rightarrow a_{10}=2+(10-1)3$

$a_{10}=2+27=\boxed{29}$

$a_3+a_5=20$

$a_3+2d+a_3+4d=30$

$2a_3+6d=30 \rightarrow 10+6d=30$

$6d=20 \rightarrow d=3$

(۹)

$a_n=a_1+(n-1)d$

$S_n=\frac{n}{2}(2a_1+(n-1)d) \rightarrow \frac{6}{2}(2a_1+5d)=9\times\frac{4}{2}(2a_1+d)$

$3(2a_1+5d)=36(2a_1+d)$

$6a_1+15d=72a_1+36d$

$21d=66a_1 \rightarrow a_1=\frac{7}{22}d$

$a_{10}=\frac{7}{22}d+9d=\frac{205}{22}d$

$a_{70}=\frac{7}{22}d+69d=\frac{1527}{22}d$

$\frac{a_{10}}{a_{70}}=\frac{\frac{205}{22}d}{\frac{1527}{22}d}=\boxed{\frac{205}{1527}}$

(۱۰)

دنباله اول:

$a_1=11 \quad a_7=35$

$a_n=a_1+(n-1)d$

$a_7=11+6d=35$

$6d=24 \rightarrow d=4$

$a_4=11+3d \rightarrow 11+12=\boxed{23}$

دنباله دوم:

$a_1=28 \quad a_{k+2}=12$

$n=k+2$: تعداد جملات دنباله

$a_n=a_1+(n-1)d \Rightarrow a_{k+2}=12$

$12=28+(k+1)d \rightarrow (k+1)d=-16$

$d=\frac{-16}{k+1}$

$a_4=28+3d$

$a_4=23 \rightarrow 28+3d=23 \rightarrow 3d=-15$

$d=-5$

شرط:

$d=\frac{-16}{k+1}=-5 \rightarrow \frac{16}{k+1}=5 \rightarrow k+1=5$

$\boxed{k=4}$: جواب