

۱, ۵, ۱۲, ۲۲

$$a_n = n^2 + \frac{n(n-1)}{2}$$

$$a_{10} = 100 + \frac{90}{2} = 145$$

۱

تفاوت اولی: ۱, ۴, ۹, ... → n^2
تفاوت دومی: ۱, ۳, ۵, ... → $\frac{n(n+1)}{2}$

$$\left. \begin{array}{l} \text{تفاوت اولی} \\ \text{تفاوت دومی} \end{array} \right\} \text{تفاوت سومی} = n^2 - \frac{n(n+1)}{2} = \frac{2n^2 - n^2 - n}{2} = \frac{n^2 - n}{2}$$

$$\frac{n^2 - n}{2} = 56 \quad n(n-1) = 110 \rightarrow n = 11$$

۲

تفاوت اولی: ۵, ۱۳, ۲۱, ... → $t_n = 5 + 8(n-1) = 8n - 3$
تفاوت دومی: ۵, ۹, ۱۳, ... → $t_n = 5 + 4(n-1) = 4n + 1$
تفاوت سومی: $8n - 3 + 4n + 1 = 12n - 2$

$$\left. \begin{array}{l} \text{تفاوت اولی} \\ \text{تفاوت دومی} \end{array} \right\} \rightarrow t_{11}: \begin{array}{l} 8 \times 11 - 3 = 85 \\ 4 \times 11 + 1 = 45 \\ 85 - 45 = 40 \end{array}$$

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$$a_n = \frac{(-1)^n}{n}$$

if $n=1 \Rightarrow -1 \rightarrow$ اولی
if $n=2 \Rightarrow \frac{1}{2} \rightarrow$ دومی
if $n=3 \Rightarrow -\frac{1}{3} \rightarrow$ سومی
if $n=4 \Rightarrow \frac{1}{4} \rightarrow$ چهارمی

$$\frac{1}{4} - (-1) = 1.25$$

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$$b_n = (-3)^n - 7n \xrightarrow{n=3} -27 - 21 = -48$$

$$-5n + 22 = -48$$

$$a_n = -5n + 22 \xrightarrow{n=k} -5k + 22$$

$$-5k + 22 = -48$$

$$-5k = -70 \rightarrow \boxed{k=14}$$

۵

$t_{10} - t_v = 12$ $t_4 - t_1 = ? \quad t_4 - (t_4 - t_0) = (t_0)$ $d = \frac{t_v - t_1}{\frac{10 - v}{r}} = 2$ $t_1 = t_4 - \Delta d \rightarrow t_4 - t_0$	6
$t_r = v \quad t_\varepsilon = 15$ $(3), v, (11), 15$ $d = \frac{t_\varepsilon - t_r}{\varepsilon - r} = \frac{15}{r} = 2$ $\begin{matrix} 3, 11 \\ \downarrow \quad \downarrow \\ +1 \quad +1 \end{matrix} \quad \begin{matrix} 19, 17, 15, 13 \\ \downarrow \quad \downarrow \quad \downarrow \\ +1 \quad +1 \quad +1 \end{matrix}$ $b_n = 1n - 5$	7
$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$ $O_{2n}: \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \rightarrow \frac{1}{2n+1}$ $\frac{1}{2n+1} < \frac{1}{n}$ $1n - 5 < 1n + 9$ $1n < 14 \quad \boxed{n = 1, 2, 3, 4, 5}$ $O_{3n}: \frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12} \rightarrow \frac{1}{n+2}$ $n < \frac{15}{2} \rightarrow n < 7.5$	8
$t_n = 4, 9, 9, 4, 1, \dots$ $\begin{matrix} 4 & 9 & 9 & 4 & 1 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ +3 & +1 & -1 & -4 & -5 \\ \hline -1 & -2 & -2 & -5 & -5 \end{matrix}$ $ra = -r \quad a = -\frac{r}{r}$ $\frac{1}{2}a + b = r \rightarrow b = \frac{15}{r}$ $t_n = -\frac{r}{r}n^2 + \frac{15}{r}n$ $ra + 2b = r \times -\frac{r}{r} + 2 \times \frac{15}{r} = -\frac{r}{r} + \frac{30}{r} = \frac{29}{r} = 29 \quad (r=1)$	9
$t_n = -4, -1, 0, \dots$ $\begin{matrix} -4 & -1 & 0 \\ \downarrow & \downarrow & \downarrow \\ +3 & +2 & +1 \\ \hline -1 & -1 & -1 \end{matrix}$ $ra = r \quad a = 1$ $\frac{1}{2}a + b = r \rightarrow b = -1$ $a + b + c = -4 \quad \begin{matrix} 1 & -1 & -4 \end{matrix}$ $t_n = n^2 - n - 4$ $a_r = 31 \quad a_\varepsilon = 51 \quad a = \frac{51 - 31}{r} = 2$ $a_n = 14, 21, 24, 31, \dots \rightarrow a_n = 5n + 11$ $5n + 11 = n^2 - n - 4 \rightarrow n^2 - 4n - 15 = 0$ $\begin{matrix} \nearrow n=9 \checkmark \\ \searrow n=-5 \times \end{matrix}$	10

$$(n-9)(n+5) \quad n = \frac{9-5}{-15} = \frac{4}{-15} = \frac{44}{-15}$$