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الف) $t_1 = (3 \times 1) - 1 = 2 \Rightarrow 2, 11, 24, 47, 74, \dots$ $t_2 = (3 \times 4) - 1 = 11$ $t_3 = (3 \times 9) - 1 = 26$ $t_4 = (3 \times 16) - 1 = 47$ $t_5 = (3 \times 25) - 1 = 74$	$t_1 = \frac{3+1}{3} = \frac{4}{3} \Rightarrow \frac{4}{3}, \frac{16}{9}, \frac{64}{27}, \frac{256}{81}, \dots$ $t_2 = \frac{4+1}{4} = \frac{5}{4}$ $t_3 = \frac{9+1}{9} = 1$ $t_4 = \frac{1}{1}$ $t_5 = \frac{11}{9}$	۱					
الف) $n=13 \Rightarrow t_{13} = \frac{(-1)^{13}}{\sqrt{13+3}} = \frac{-1}{\sqrt{16}} = -\frac{1}{4}$ ب) $n=13 \Rightarrow t_{13} = (2 \times 13) - \left[\frac{13}{4} \right] = 26 - 3 = 23$ $\left[\frac{13}{4} \right] = 3$		۲					
الف) $2n-10 < 0 \Rightarrow 2n < 10 \Rightarrow n < 5 \Rightarrow n = 1, 2, 3, 4 \Rightarrow t_1, t_2, t_3, t_4$ ب) $n^2 - 14n + 40 < 0 \Rightarrow \frac{n}{(n-10)} \frac{4}{(n-4)} \Rightarrow$ <table style="margin: 0 10px;"> <tr><td>+</td><td>0</td><td>-</td><td>0</td><td>+</td></tr> </table> $\Rightarrow 5, 6, 7, 8, 9 \Rightarrow t_5, t_6, t_7, t_8, t_9$ صورتی و معکوس	+	0	-	0	+		۳
+	0	-	0	+			
الف) $2n-14 \leq 0 \Rightarrow 2n \leq 14 \Rightarrow n \leq 7 \Rightarrow n = 1, 2, 3, 4, 5, 6, 7 \Rightarrow$ ب) $n^2 - 12n + 27 \leq 0 \Rightarrow \frac{n}{(n-9)} \frac{3}{(n-3)} \Rightarrow$ <table style="margin: 0 10px;"> <tr><td>+</td><td>0</td><td>-</td><td>0</td><td>+</td></tr> </table> $\Rightarrow n = 3, 4, 5, 6, 7, 8, 9 \Rightarrow$	+	0	-	0	+		۴
+	0	-	0	+			
$t_n = an + b$ $t_4 \Rightarrow (4a + b = 11) \times (-1) \Rightarrow -4a - b = -11$ $t_9 \Rightarrow 9a + b = 22$ $\begin{cases} -4a - b = -11 \\ 9a + b = 22 \end{cases} \xrightarrow{a=3} 27 + b = 22 \rightarrow b = -5$ $5a = 15 \rightarrow a = 3$ $\Rightarrow n = 3 \rightarrow t_3 = (3 \times 3) - 5 = 4$							

الف) $n^2 - 4n + 2 \xrightarrow{a \geq 0} \min \Rightarrow \left[\begin{aligned} -\frac{b}{2a} &= -\frac{-4}{2} = 2 \\ 4 - 16 + 2 &= -10 \end{aligned} \right]$ کمترین مقدار -10 است و برای $n=2$

ب) $n^2 + 4n + 1 \xrightarrow{a \geq 0} \min \Rightarrow \left[\begin{aligned} -\frac{b}{2a} &= -\frac{4}{2} = -2 \rightarrow -2 \notin \mathbb{N} \end{aligned} \right]$ پس $t_1 = 1 + 4 + 1 = 6$
تقریباً مقدار برای $n=1$
پس $t_2 = 4 + 16 + 1 = 21$
پس $t_3 = 9 + 36 + 1 = 46$

$t_n = -4, -1, 1, 4, \dots \rightsquigarrow$ تفاوت a تفاوت b
 $\rightsquigarrow an^2 + bn + c$
 $\rightsquigarrow a = \frac{1}{2} = 1, c = 1 \rightarrow b = -4$
 $\Rightarrow a_n = n^2 - 4n + 1 \rightarrow a_{10} = 100 - 40 + 1 = 61$

$$\begin{aligned} -4 &= 4 + 1b + c \\ (-4 = 1 + b + c) \times (-1) \\ -4 &= 1 + b + c \rightarrow b = -4 \\ c &= 1 \end{aligned}$$

$t_n = -4, -1, 1, 4, \dots \rightsquigarrow an^2 + bn + c \rightsquigarrow 2n^2 + bn + c$
 $\rightsquigarrow 2n^2 + bn - 4 \xrightarrow{n=1} 2 + b - 4 = -4 \rightarrow b = -2$
 $\rightsquigarrow t_n = 2n^2 - 2n - 4$
 $\left[\begin{aligned} a &= 2 \\ b &= -2 \\ c &= -4 \end{aligned} \right]$

حالت اول $\rightarrow 2a + 1 \leq a + 4 \rightarrow a \leq 3 \quad \underline{a \in (-\infty, 3]}$
 حالت دوم $\rightarrow$ (استقراف متناهی) $\rightarrow a + 4 = 2a + 1 \rightarrow a = 3$
 حالت سوم $\rightarrow$ (استقراف نامتناهی) $\rightarrow 2a + 1 > a + 4 \rightarrow a > 3$
} $a \geq 3 \quad \underline{a \in [3, +\infty)}$

الف) $\frac{99-3}{3} + 1 = 33$

ب) $\frac{100-5}{5} + 1 = 20$

ج) $\frac{90-15}{15} + 1 = 7$

د) $n(A \cup B) = n(A) + n(B) - n(A \cap B) \rightarrow 33 + 20 - 7 = 46$