

۱) $y = x^2 - 5$ $x^2 = y + 5$
 $x = \sqrt{y+5} \Rightarrow y \geq -5$ $R_f = [-5, +\infty)$

۲) $y = x^3 + 1$ $x^3 = y - 1$
 $x = \sqrt[3]{y-1} \Rightarrow R_f = \mathbb{R}$

الف) $y = x^2 - 4x + 2$ $x^2 - 4x(4-y) = 0 \Rightarrow \Delta \geq 0$ $R_f = [1, +\infty)$
 $b^2 - 4ac$
 $(-4)^2 - 4(1)(4-y) = 4y - 12$
 $4y - 12 \geq 0 \Rightarrow y \geq 3$

ب) $y = x^2 - 5x + 1$ $\Delta = b^2 - 4ac$
 $x^2 - 5x + (1-y) = 0$ $25 - 4(1)(1-y) = 4y + 21$
 $4y + 21 \geq 0 \Rightarrow y \geq -\frac{21}{4}$

الف) $y = \frac{x^2 + 3}{x^2 - 2}$ $y(x^2 - 2) = x^2 + 3$
 $y x^2 - 2x = x^2 + 3$
 $y x^2 - x^2 - 2x - 3 = 0$
 $x^2(y-1) - 2x - 3 = 0$
 $\Delta \geq 0$
 $4 - 4(y-1)(-3) \geq 0$
 $12y - 12 \geq 0 \Rightarrow y \geq 1$

ب) $y = \frac{2|x| + 1}{|x| - 4}$ $y(|x| - 4) = 2|x| + 1$
 $y|x| - 4y = 2|x| + 1$
 $y|x| - 2|x| = 4y + 1$
 $|x|(y-2) = 4y + 1$
 $|x| = \frac{4y+1}{y-2} \geq 0$

$y x^2 - 4x y = 1$
 $x^2(y) - 4y(x) - 1 = 0$
 $x = \frac{-b \pm \sqrt{\Delta}}{2a}$
 $\Delta = b^2 - 4ac$
 $(-4y)^2 - 4(y)(-1) = 16y^2 + 4y$
 $16y^2 + 4y \geq 0 \Rightarrow y \geq -\frac{1}{4}$

الف) $y = x^2 - 4x + 2$ $\min \left\{ \frac{-b}{2a} = \frac{4}{2} = 2 \right\}$ $R_f = [2, +\infty)$
 $x^2 - 4x + 2 = 0$ $x = 2$ $y = 2^2 - 4(2) + 2 = -2$

ب) $y = -x^2 + 4x + 2$ $\max \left\{ \frac{-b}{2a} = \frac{-4}{-2} = 2 \right\}$ $R_f = (-\infty, 6]$
 $-x^2 + 4x + 2 = 0$ $x = 2$ $y = -2^2 + 4(2) + 2 = 6$

الف) $y = \sqrt{x^2 - 6x + 2}$ $\begin{cases} \frac{-b}{2a} = \frac{3}{1} = 3 \\ \frac{4-36+2}{4} = -\frac{11}{4} \end{cases}$ $R_f = \sqrt{[1, +\infty)}$ $R_f = [0, +\infty)$

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ب) $y = \sqrt{-x^2 + 8x + 10}$ $\begin{cases} \frac{-b}{2a} = \frac{-4}{-1} = 4 \\ \frac{-16+32+10}{-4} = 15 \end{cases}$ $R_f = \sqrt{(-\infty, 15]}$ $R_f = [0, \sqrt{15}]$

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الف) $y = x^4 + 13x^2 + 2x + 1$ $\Rightarrow R_f = (-\infty, +\infty) = \mathbb{R}$

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ب) $y = \sqrt{x^4 + 6x^2 + 6x + 1}$ $\Rightarrow R_f = \sqrt{\mathbb{R}} \Rightarrow [0, +\infty)$

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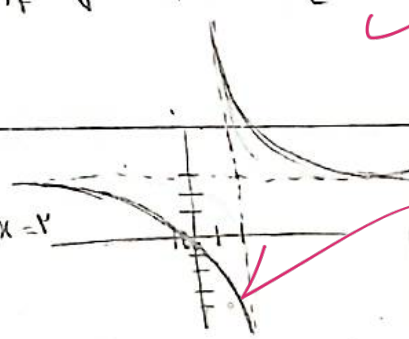
الف) $y = \frac{3x+1}{x-2}$ $R_f = \mathbb{R} - \{2\}$

ب) $y = \frac{5x+1}{\sqrt{x+3}}$ $R_f = \sqrt{\mathbb{R} - \{-3\}} \Rightarrow [0, +\infty) - \{2\}$

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الف) $y = \frac{3x+1}{x-2}$

$x=0 \Rightarrow \frac{1}{-2}$
 $y=0 \Rightarrow \frac{1}{3}$



مخارج قسمة $\leftarrow$ رأسية مخارج $\leftarrow$

مخارج افقي $\leftarrow$ $y = \frac{a}{c}$

$y = \frac{3}{1}$

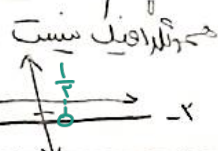
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ب) $y = \frac{4x-2}{1-2x}$

$\Rightarrow \frac{ad+bc}{f^2} = \frac{4(-2)+(-2)(-1)}{1^2} = -6$

$\frac{-2(4x+1)}{-2x+1} = -2$

$y = -2$



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الف) $y = \cos^2 x + \frac{1}{\cos^2 x}$

$\cos^2 x > 0$

$\Rightarrow [1, +\infty)$

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ب) $y = \sqrt{\frac{x^2+1}{x}}$

$\Rightarrow \sqrt{\frac{x^2}{x} + \frac{1}{x}} \Rightarrow \sqrt{x + \frac{1}{x}} \Rightarrow$

$x > 0 \quad [\frac{1}{x}, +\infty)$

$x < 0 \quad (-\infty, -2]$

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سوال ۲

$$الف) y = x^2 - 2x + 4 \rightarrow y = (x-1)^2 + 3$$

$$(x-1)^2 = y-3 \rightarrow x = \pm \sqrt{y-3} + 1 \rightarrow y \geq 3$$

$$ب) y = x^2 - 2x + 1 \rightarrow y = (x-1)^2 - \frac{1}{2}$$

$$x - \frac{1}{2} = \pm \sqrt{y + \frac{1}{2}} \rightarrow x = \pm \sqrt{y + \frac{1}{2}} + \frac{1}{2}$$

$$y \geq -\frac{1}{2}$$

سوال ۳ قسمت الف

$$y = \frac{x^2 + 3}{x^2 - 2}$$

$$x^2 y - 2y = x^2 + 3$$

$$x^2 (y-1) = 2y+3 \rightarrow x = \pm \sqrt{\frac{2y+3}{y-1}}$$

$$\frac{-\frac{3}{2}}{+1 - 1+}$$

$$Ry = (-\infty, -\frac{3}{2}] \cup (1, +\infty)$$